Online Appendices for
"The Time Variation in Risk Appetite and Uncertainty"

A The state variables

A.1 Matrix representation of the state variables

In this section, we show the matrix representation of the system of ten state variables in this economy. The ten state variables, as introduced in Section 3, are as follows,

$$\boldsymbol{Y_t} = [\theta_t, p_t, n_t, \pi_t, l_t, g_t, \kappa_t, \eta_t, lp_t, q_t]',$$

where $\{p_t, n_t\}$ denote the upside uncertainty factor and the downside uncertainty factor, as latent variables extracted from the system of output growth (i.e., change in log real industrial production index); π_t represents the inflation rate; l_t represents the log of corporate loss rate; g_t represents the log change in real earnings; κ_t represents the log consumption-earnings ratio; η_t represents the log dividend payout ratio; lp_t represents the cash flow uncertainty factor, as the latent variable extracted from the system of corporate loss rate l_t; q_t represents the latent risk aversion of the economy. The state variables have the following matrix representation:

$$\boldsymbol{Y_{t+1}} = \boldsymbol{\mu} + \boldsymbol{A}\boldsymbol{Y_t} + \boldsymbol{\Sigma}\boldsymbol{\omega_{t+1}}, \tag{A.1}$$

where $\boldsymbol{\omega_{t+1}} = [\omega_{p,t+1}, \omega_{n,t+1}, \omega_{\pi,t+1}, \omega_{lp,t+1}, \omega_{ln,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}, \omega_{q,t+1}]$ (9×1) is a vector comprised of eight independent shocks in the economy. Among the nine shocks, $\{\omega_{\pi,t+1}, \omega_{ln,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}\}$ shocks are homoskedastic. The conditional variance, skewness and higher-order moments of the following four centered gamma shocks—$\omega_{p,t+1}$, $\omega_{n,t+1}$, $\omega_{lp,t+1}$, and $\omega_{q,t+1}$ —are assumed to be proportional to p_t, n_t, lp_t, and q_t respectively. The underlying distributions for the rest four shocks are assumed to be Gaussian with unit standard deviation.

The constant matrices are defined implicitly,

$$\boldsymbol{\mu} = \begin{bmatrix} (1-\rho_\theta)\bar{\theta} - m_p\bar{p} - m_n\bar{n} \equiv \theta_0 \\ (1-\rho_p)\bar{p} \equiv p_0 \\ (1-\rho_n)\bar{n} \equiv n_0 \\ \pi_0 \\ l_0 \\ g_0 \\ \kappa_0 \\ \eta_0 \\ lp_0 \\ q_0 \end{bmatrix}, \tag{A.2}$$

$$\boldsymbol{A} = \begin{bmatrix} \rho_\theta & m_p & m_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{\pi\theta} & \rho_{\pi p} & \rho_{\pi n} & \rho_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{lp} & \rho_{ln} & 0 & \rho_{ll} & 0 & 0 & 0 & 0 & 0 \\ \rho_{g\theta} & \rho_{gp} & \rho_{gn} & \rho_{gl} & 0 & \rho_{gg} & 0 & 0 & \rho_{glp} & 0 \\ \rho_{\kappa\theta} & \rho_{\kappa p} & \rho_{\kappa n} & \rho_{\kappa l} & 0 & 0 & \rho_{\kappa\kappa} & 0 & \rho_{\kappa lp} & 0 \\ \rho_{\eta\theta} & \rho_{\eta p} & \rho_{\eta n} & \rho_{\eta l} & 0 & 0 & 0 & \rho_{\eta\eta} & \rho_{\eta lp} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{lp} & 0 \\ 0 & \rho_{qp} & \rho_{qn} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{qq} \end{bmatrix}, \tag{A.3}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{\theta p} & -\sigma_{\theta n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{pp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{nn} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi p} & \sigma_{\pi n} & \sigma_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{lp} & \sigma_{ln} & 0 & \sigma_{llp} & -\sigma_{lln} & 0 & 0 & 0 & 0 \\ \sigma_{gp} & \sigma_{gn} & 0 & \sigma_{glp} & \sigma_{gln} & \sigma_{gg} & 0 & 0 & 0 \\ \sigma_{\kappa p} & \sigma_{\kappa n} & 0 & \sigma_{\kappa lp} & \sigma_{\kappa ln} & 0 & \sigma_{\kappa\kappa} & 0 & 0 \\ \sigma_{\eta p} & \sigma_{\eta n} & 0 & \sigma_{\eta lp} & \sigma_{\eta ln} & 0 & 0 & \sigma_{\eta\eta} & 0 \\ 0 & 0 & 0 & \sigma_{lplp} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{qp} & \sigma_{qn} & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{qq} \end{bmatrix}. \tag{A.4}$$

Given the moment generating functions (mgf) of gamma and Gaussian distributions, we show that the model is affine, $\forall \boldsymbol{\nu} \in \mathbb{R}^{10}$,

$$\begin{aligned} M_Y(\boldsymbol{\nu}) := E_t\left[\exp(\boldsymbol{\nu'Y_{t+1}})\right] &= \exp(\boldsymbol{\nu'\mu} + \boldsymbol{\nu'AY_t})E_t\left[\exp(\boldsymbol{\nu'\Sigma\omega_{t+1}})\right] \\ &= \exp\left[\boldsymbol{\nu'S_0} + \frac{1}{2}\boldsymbol{\nu'S_1\Sigma^{other}S_1'\nu} + \boldsymbol{f_S(\nu)Y_t} + S_2(\boldsymbol{\nu})v_n\right], \end{aligned} \tag{A.5}$$

where $\boldsymbol{S_0} = \boldsymbol{\mu}$ (10×1),

$$\boldsymbol{S_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{A.6}$$

$$\boldsymbol{\Sigma^{other}} = \begin{bmatrix} \sigma^2_{\pi\pi} & \sigma_{\pi g} & \sigma_{\pi\kappa} & \sigma_{\pi\eta} \\ \sigma_{g\pi} & \sigma^2_{gg} & \sigma_{g\kappa} & \sigma_{g\eta} \\ \sigma_{\kappa\pi} & \sigma_{\kappa g} & \sigma^2_{\kappa\kappa} & \sigma_{\kappa\eta} \\ \sigma_{\eta\pi} & \sigma_{\eta g} & \sigma_{\eta\kappa} & \sigma^2_{\eta\eta} \end{bmatrix} \text{ (cov-var matrix of } \{\omega_\pi, \omega_g, \omega_\kappa, \omega_\eta\}), \tag{A.7}$$

$$\boldsymbol{f_S(\nu)} = \boldsymbol{\nu'A} + \begin{bmatrix} 0 \\ -\sigma_p(\boldsymbol{\nu}) - \ln(1-\sigma_p(\boldsymbol{\nu})) \\ -\sigma_n(\boldsymbol{\nu}) - \ln(1-\sigma_n(\boldsymbol{\nu})) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma_{lp}(\boldsymbol{\nu}) - \ln(1-\sigma_{lp}(\boldsymbol{\nu})) \\ -\sigma_q(\boldsymbol{\nu}) - \ln(1-\sigma_q(\boldsymbol{\nu})) \end{bmatrix}', \tag{A.8}$$

$$S_2(\boldsymbol{\nu}) = -\sigma_{ln}(\boldsymbol{\nu}) - \ln(1-\sigma_{ln}(\boldsymbol{\nu})), \tag{A.9}$$

$$\sigma_p(\boldsymbol{\nu}) = \boldsymbol{\nu'\Sigma_{\bullet 1}}, \tag{A.10}$$

$$\sigma_n(\boldsymbol{\nu}) = \boldsymbol{\nu'\Sigma_{\bullet 2}}, \tag{A.11}$$

$$\sigma_{lp}(\boldsymbol{\nu}) = \boldsymbol{\nu'\Sigma_{\bullet 4}}, \tag{A.12}$$

$$\sigma_{ln}(\boldsymbol{\nu}) = \boldsymbol{\nu'\Sigma_{\bullet 5}}, \tag{A.13}$$

$$\sigma_q(\boldsymbol{\nu}) = \boldsymbol{\nu'\Sigma_{\bullet 9}}, \tag{A.14}$$

where $\boldsymbol{M_{\bullet j}}$ denotes the j-th column of the matrix M.

A.2 Consumption growth

Consumption growth in this economy is endogenous defined and can be expressed in an affine function:

$$\Delta c_{t+1} = g_{t+1} + \Delta\kappa_{t+1} \tag{A.15}$$

$$= c_0 + \boldsymbol{c_2'Y_t} + \boldsymbol{c_1'\Sigma\omega_{t+1}}, \tag{A.16}$$

(A.17)

where $c_0 = g_0 + \kappa_0$, $\boldsymbol{c_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}'$, and

$$\boldsymbol{c_2} = \begin{bmatrix} \rho_{g\theta} + \rho_{\kappa\theta} \\ \rho_{gp} + \rho_{\kappa p} \\ \rho_{gn} + \rho_{\kappa n} \\ 0 \\ 0 \\ \rho_{gg} \\ \rho_{\kappa\kappa} - 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{A.18}$$

B Asset Pricing

In this section, we solve the model analytically. First, given consumption growth and changes in risk aversion, the log of real pricing kernel of the economy is derived as an affine function of the state variables. Next, we show that asset prices of claims on cash flows from three different asset markets can be expressed in (quasi) affine equations. The model is solved using the non-arbitrage condition. The goal of this section is to derive the analytical solutions for the expected excess returns, the physical variance of asset returns and the risk-neutral variance of asset returns in closed forms. The implied moments are crucial for the estimation procedure.

B.1 The real pricing kernel

The log real pricing kernel for this economy is given by,

$$m_{t+1} = \ln(\beta) - \gamma\Delta c_{t+1} + \gamma\Delta q_{t+1} \tag{B.1}$$
$$= m_0 + \boldsymbol{m_2'Y_t} + \boldsymbol{m_1'\Sigma\omega_{t+1}}, \tag{B.2}$$

where $m_0 = \ln(\beta) + \gamma(q_0 - g_0 - \kappa_0)$, $\boldsymbol{m_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\gamma & -\gamma & 0 & 0 & \gamma \end{bmatrix}'$, and

$$\boldsymbol{m_2} = \begin{bmatrix} \gamma(-\rho_{g\theta} - \rho_{\kappa\theta}) \\ \gamma(\rho_{qp} - \rho_{gp} - \rho_{\kappa p}) \\ \gamma(\rho_{qn} - \rho_{gn} - \rho_{\kappa n}) \\ 0 \\ 0 \\ -\gamma\rho_{gg} \\ -\gamma(\rho_{\kappa\kappa} - 1) \\ 0 \\ 0 \\ \gamma(\rho_{qq} - 1) \end{bmatrix}. \tag{B.3}$$

As a result, the moment generating function of the real pricing kernel is, $\forall \nu \in \mathbb{R}$,

$$\begin{aligned} E_t\left[\exp(\nu m_{t+1})\right] = & \exp\left[\nu m_0 + \nu \boldsymbol{m_2'Y_t}\right] \\ \cdot & \exp\left\{\left[-\nu\sigma_p(\boldsymbol{m_1}) - \ln\left(1 - \nu\sigma_p(\boldsymbol{m_1})\right)\right] p_t + \left[-\nu\sigma_n(\boldsymbol{m_1}) - \ln\left(1 - \nu\sigma_n(\boldsymbol{m_1})\right)\right] n_t\right\} \\ \cdot & \exp\left\{\left[-\nu\sigma_{lp}(\boldsymbol{m_1}) - \ln\left(1 - \nu\sigma_{lp}(\boldsymbol{m_1})\right)\right] lp_t + \left[-\nu\sigma_q(\boldsymbol{m_1}) - \ln\left(1 - \nu\sigma_q(\boldsymbol{m_1})\right)\right] q_t\right\} \\ \cdot & \exp\left\{\left[-\nu\sigma_{ln}(\boldsymbol{m_1}) - \ln\left(1 - \nu\sigma_{ln}(\boldsymbol{m_1})\right)\right] v_n + \frac{1}{2}\nu^2\left[\boldsymbol{m_1'S_1\Sigma^{other}S_1'm_1}\right]\right\}, \end{aligned} \tag{B.4}$$

where m_0, $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, $\boldsymbol{S_1}$, and $\boldsymbol{\Sigma^{other}}$ are constant matrices defined earlier, and

$$\sigma_p(\boldsymbol{m_1}) = \boldsymbol{m_1'\Sigma_{\bullet 1}}, \tag{B.5}$$
$$\sigma_n(\boldsymbol{m_1}) = \boldsymbol{m_1'\Sigma_{\bullet 2}}, \tag{B.6}$$
$$\sigma_{lp}(\boldsymbol{m_1}) = \boldsymbol{m_1'\Sigma_{\bullet 4}}, \tag{B.7}$$
$$\sigma_{ln}(\boldsymbol{m_1}) = \boldsymbol{m_1'\Sigma_{\bullet 5}}, \tag{B.8}$$
$$\sigma_q(\boldsymbol{m_1}) = \boldsymbol{m_1'\Sigma_{\bullet 9}}. \tag{B.9}$$

Accordingly, the model-implied short rate rf_t is,

$$
\begin{aligned}
rf_t &= -\ln\left\{E_t\left[\exp(m_{t+1})\right]\right\} && \text{(B.10)}\\
&= -m_0 - \boldsymbol{m_2'Y_t} && \text{(B.11)}\\
&+ \left[\sigma_p(\boldsymbol{m_1}) + \ln\left(1-\sigma_p(\boldsymbol{m_1})\right)\right]p_t + \left[\sigma_n(\boldsymbol{m_1}) + \ln\left(1-\sigma_n(\boldsymbol{m_1})\right)\right]n_t && \text{(B.12)}\\
&+ \left[\sigma_{lp}(\boldsymbol{m_1}) + \ln\left(1-\sigma_{lp}(\boldsymbol{m_1})\right)\right]lp_t + \left[\sigma_q(\boldsymbol{m_1}) + \ln\left(1-\sigma_q(\boldsymbol{m_1})\right)\right]q_t && \text{(B.13)}\\
&+ \left[\sigma_{ln}(\boldsymbol{m_1}) + \ln\left(1-\sigma_{ln}(\boldsymbol{m_1})\right)\right]v_n - \frac{1}{2}\left[\boldsymbol{m_1'S_1\Sigma^{other}S_1'm_1}\right], && \text{(B.14)}\\
&= rf_0 + \boldsymbol{rf_2'Y_t}. && \text{(B.15)}
\end{aligned}
$$

To price nominal assets, we define the nominal pricing kernel, $\widetilde{m}_{t+1}$, which is a simple transformation of the log real pricing kernel, m_{t+1},

$$
\begin{aligned}
\widetilde{m}_{t+1} &= m_{t+1} - \pi_{t+1}, && \text{(B.16)}\\
&= \widetilde{m}_0 + \boldsymbol{\widetilde{m}_2'Y_t} + \boldsymbol{\widetilde{m}_1'\Sigma\omega_{t+1}}, && \text{(B.17)}
\end{aligned}
$$

where $\widetilde{m}_0 = m_0 - \pi_0$, $\boldsymbol{\widetilde{m}_1} = \boldsymbol{m_1} - \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}'$, and

$$
\boldsymbol{\widetilde{m}_2} = \boldsymbol{m_2} - \begin{bmatrix}\rho_{\pi\theta}\\ \rho_{\pi p}\\ \rho_{\pi n}\\ \rho_{\pi\pi}\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}. \qquad \text{(B.18)}
$$

The nominal risk free rate $\widetilde{rf}_t$ is defined as $-\ln\left\{E_t\left[\exp(\widetilde{m}_{t+1})\right]\right\}$.

B.2 Valuation ratio

It is a crucial step in this paper to show that asset prices are (quasi) affine functions of the state variables.

Defaultable Nominal Bonds In the paper, we assume that a one period nominal bond faces a fractional (logarithmic) loss of l_t. Given the structures assumed for l_t and π_t and the model-implied log pricing kernel, the price-coupon ratio of the one-period defaultable bond portfolio is

$$
\begin{aligned}
PC_t^1 &= E_t\left[\exp\left(\widetilde{m}_{t+1} - l_{t+1}\right)\right] && \text{(B.19)}\\
&= \exp\left(b_0^1 + \boldsymbol{b_1^{1\prime}Y_t}\right), && \text{(B.20)}
\end{aligned}
$$

where b_0^1 and $\boldsymbol{b_1^{1\prime}}$ are implicitly defined. Consider next a portfolio of multi-period zero-coupon defaultable bonds with a promised terminal payment of C at period $(t+N)$. As for the N-period bond, the actual payment will be less than or equal to the promised payment, and the ex-post nominal payoff can be expressed as $\exp\left(c - l_{t+N}\right)$. We ignore the possibility of early default or prepayment. Then, the price-coupon ratio of this bond at period $(t+N-1)$, one period before maturity, PC_{t+N-1}^1, is $\exp\left(b_0^1 + b_1^{1\prime}Y_{t+N-1}\right)$. Given the Euler equation and the law of iterated expectations, it then follows by induction that all earlier dated zero-coupon nominally defaultable corporate bond (maturing in N period) prices are similarly affine in the state variables, in particular:

$$
\begin{aligned}
PC_t^N &= E_t\left[\widetilde{M}_{t+1}PC_{t+1}^{N-1}\right],\\
&= \exp\left(b_0^N + \boldsymbol{b_1^{N\prime}Y_t}\right). && \text{(B.21)}
\end{aligned}
$$

Therefore, the assumed zero-coupon structure of the payments before maturity implies that the unexpected returns to this portfolio are exactly linearly spanned by the shocks to $\boldsymbol{Y_t}$.

Equity It is especially not obvious for equity price-dividend ratio, of which we provide proofs below. First, we rewrite the real dividend growth in a general matrix expression:

$$\begin{aligned} \Delta d_{t+1} &= g_{t+1} + \Delta\eta_{t+1} \\ &= h_0 + \boldsymbol{h_2'Y_t} + \boldsymbol{h_1'\Sigma\omega_{t+1}}, \end{aligned} \tag{B.22}$$

where $h_0 = g_0 + \eta_0$, $\boldsymbol{h_1} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0\end{bmatrix}'$, and

$$\boldsymbol{h_2} = \begin{bmatrix} \rho_{g\theta} + \rho_{\eta\theta} \\ \rho_{gp} + \rho_{\eta p} \\ \rho_{gn} + \rho_{\eta n} \\ 0 \\ 0 \\ \rho_{gg} \\ 0 \\ \rho_{\eta\eta} - 1 \\ 0 \\ 0 \end{bmatrix}. \tag{B.23}$$

The price-dividend ratio, $PD_t = E_t\left[M_{t+1}\left(\frac{P_{t+1}+D_{t+1}}{D_t}\right)\right]$, can be rewritten as,

$$PD_t = \sum_{n=1}^{\infty} E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j}\right)\right]. \tag{B.24}$$

Let F_t^n denote the n-th term in the summation:

$$F_t^n = E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j}\right)\right], \tag{B.25}$$

and $F_t^n D_t$ is the price of zero-coupon equity that matures in n periods.

To show that equity price is an approximate affine function of the state variables, we first prove that $F_t^n (\forall n \geq 1)$ is exactly affine using induction. First, when $n = 1$,

$$\begin{aligned} F_t^1 &= E_t\left[\exp\left(m_{t+1} + \Delta d_{t+1}\right)\right] \\ &= E_t\left\{\exp\left[(m_0 + h_0) + \boldsymbol{(m_2' + h_2')Y_t} + \boldsymbol{(m_1' + h_1')\Sigma\omega_{t+1}}\right]\right\} \\ &= \exp\left[(m_0 + h_0) + \boldsymbol{(m_2' + h_2')Y_t}\right] \\ &\cdot \exp\left\{\left[-\sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1})\right)\right] p_t + \left[-\sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1})\right)\right] n_t\right\} \\ &\cdot \exp\left\{\left[-\sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1})\right)\right] lp_t + \left[-\sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1})\right)\right] q_t\right\} \\ &\cdot \exp\left\{\left[-\sigma_{ln}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_{ln}(\boldsymbol{m_1} + \boldsymbol{h_1})\right)\right] v_n + \frac{1}{2}\left[\boldsymbol{(m_1' + h_1')S_1\Sigma^{other}S_1'(m_1 + h_1)}\right]\right\} \\ &= \exp\left(e_0^1 + \boldsymbol{e_1^{1\prime}Y_t}\right), \end{aligned} \tag{B.26}$$

where m_0, $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, h_0, $\boldsymbol{h_1}$, $\boldsymbol{h_2}$, $\boldsymbol{S_1}$, and $\boldsymbol{\Sigma^{other}}$ are constant matrices defined earlier, and

$$\sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{\Sigma_{\bullet 1}}, \tag{B.27}$$

$$\sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{\Sigma_{\bullet 2}}, \tag{B.28}$$

$$\sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{\Sigma_{\bullet 4}}, \tag{B.29}$$

$$\sigma_{ln}(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{\Sigma_{\bullet 5}}, \tag{B.30}$$

$$\sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) = (\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{\Sigma_{\bullet 9}}, \tag{B.31}$$

and $e_0^1 = m_0 + h_0 + \left[-\sigma_{ln}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_{ln}(\boldsymbol{m_1} + \boldsymbol{h_1})\right)\right] v_n + \frac{1}{2}\left[(\boldsymbol{m_1'} + \boldsymbol{h_1'})\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S_1'}(\boldsymbol{m_1} + \boldsymbol{h_1})\right]$, and

$$\boldsymbol{e_1^1} = \boldsymbol{m_2} + \boldsymbol{h_2} + \begin{bmatrix} 0 \\ -\sigma_p \boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_p(\boldsymbol{m_1} + \boldsymbol{h_1})\right) \\ -\sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_n(\boldsymbol{m_1} + \boldsymbol{h_1})\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_{lp}(\boldsymbol{m_1} + \boldsymbol{h_1})\right) \\ -\sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1}) - \ln\left(1 - \sigma_q(\boldsymbol{m_1} + \boldsymbol{h_1})\right) \end{bmatrix}. \tag{B.32}$$

Now, suppose that the $(n-1)$-th term $F_t^{n-1} = \exp\left(e_0^{n-1} + \boldsymbol{e_1^{n-1\prime}}\boldsymbol{Y_t}\right)$, then

$$\begin{aligned} F_t^n &= E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j}\right)\right] \\ &= E_t\left\{E_{t+1}\left[\exp(m_{t+1} + \Delta d_{t+1})\exp\left(\sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1}\right)\right]\right\} \\ &= E_t\left\{\exp(m_{t+1} + \Delta d_{t+1})\underbrace{E_{t+1}\left[\exp\left(\sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1}\right)\right]}_{F_{t+1}^{n-1}}\right\} \\ &= E_t\left[\exp(m_{t+1} + \Delta d_{t+1})\exp\left(e_0^{n-1} + \boldsymbol{e_1^{n-1\prime}}\boldsymbol{Y_{t+1}}\right)\right] \\ &= \exp\left(e_0^n + \boldsymbol{e_1^{n\prime}}\boldsymbol{Y_t}\right), \end{aligned} \tag{B.33}$$

where e_0^n and $\boldsymbol{e_1^{n\prime}}$ are defined implicitly.

Hence, the price-dividend ratio is approximately affine:

$$\begin{aligned} PD_t &= \sum_{n=1}^{\infty} E_t\left[\exp\left(\sum_{j=1}^{n} m_{t+j} + \Delta d_{t+j}\right)\right] \\ &= \sum_{n=1}^{\infty} F_t^n \\ &= \sum_{n=1}^{\infty} \exp\left(e_0^n + \boldsymbol{e_1^{n\prime}}\boldsymbol{Y_t}\right). \end{aligned} \tag{B.34}$$

■

B.3 Log asset returns

Log return of zero-coupon nominally defautable corporate bonds maturing at $t+N$

Given the exact exponential affine expression of the valuation ratio of this asset (see derivations above), the log return can be derived an approximate linear closed form:

$$\begin{aligned} \widetilde{r}_{t+1}^{cb,N} &= \ln\left(\frac{PC_{t+1}^{N-1} + 1}{PC_t^N}\right)\ln\left(\frac{C}{C}\right)\ln\left(\Pi_{t+1}\right) \\ &= \pi_{t+1} + \ln\left[\frac{1 + \exp\left(b_0^{N-1} + \boldsymbol{b_1^{N-1\prime}}\boldsymbol{Y_{t+1}}\right)}{\exp\left(b_0^N + \boldsymbol{b_1^{N\prime}}\boldsymbol{Y_t}\right)}\right] \\ &\approx \pi_{t+1} + \text{const.} + \frac{\exp\left(b_0^{N-1} + \boldsymbol{b_1^{N-1\prime}}\bar{\boldsymbol{Y}}\boldsymbol{b_1^{N-1\prime}}\right)}{\frac{1+\exp\left(b_0^{N-1} + \boldsymbol{b_1^{N-1\prime}}\bar{\boldsymbol{Y}}\right)}{\exp\left(b_0^N + \boldsymbol{b_1^{N\prime}}\bar{\boldsymbol{Y}}\right)}}\boldsymbol{Y_{t+1}} - \boldsymbol{b_1^{N\prime}}\boldsymbol{Y_t} \end{aligned}$$

$$= \widetilde{\xi}_0^{cb} + \widetilde{\boldsymbol{\xi}}_{\mathbf{1}}^{\boldsymbol{cb}\prime} \boldsymbol{Y_t} + \widetilde{\boldsymbol{r}}^{\boldsymbol{cb}\prime} \boldsymbol{\Sigma} \boldsymbol{\omega_{t+1}}, \tag{B.35}$$

where $\widetilde{r}_{t+1}^{cb}$ is the log *nominal* return of corporate bond from t to $t+1$, $\widetilde{\xi}_0^{cb}$ is constant, $\widetilde{\boldsymbol{\xi}}_{\mathbf{1}}^{\boldsymbol{cb}}$ is a vector of state vector coefficients, and $\widetilde{\boldsymbol{r}}^{\boldsymbol{cb}}$ is a vector of shock coefficients. Thus, this step involves linear approximation.

Log nominal equity return We apply first-order Taylor approximations to the log nominal equity return, and obtain a linear system,

$$\begin{aligned}
\widetilde{r}_{t+1}^{eq} &= \ln\left(\frac{P_{t+1}+D_{t+1}}{P_t}\Pi_{t+1}\right) \\
&= \ln\left(\frac{PD_{t+1}+1}{PD_t}\right)\ln\left(\frac{D_{t+1}}{D_t}\right)\ln\left(\Pi_{t+1}\right) \\
&= \Delta d_{t+1} + \pi_{t+1} + \ln\left[\frac{1+\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\boldsymbol{Y_{t+1}}\right)}{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\boldsymbol{Y_t}\right)}\right] \\
&\approx \Delta d_{t+1} + \pi_{t+1} + \text{const.} + \frac{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\bar{\boldsymbol{Y}}\right)\boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}}{\frac{1+\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\bar{\boldsymbol{Y}}\right)}{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\bar{\boldsymbol{Y}}\right)}}\boldsymbol{Y_{t+1}} - \frac{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\bar{\boldsymbol{Y}}\right)\boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}}{\sum_{n=1}^{\infty}\exp\left(e_0^n + \boldsymbol{e}_{\mathbf{1}}^{\boldsymbol{n}\prime}\bar{\boldsymbol{Y}}\right)}\boldsymbol{Y_t} \\
&= \widetilde{\xi}_0^{eq} + \widetilde{\boldsymbol{\xi}}_{\mathbf{1}}^{\boldsymbol{eq}\prime}\boldsymbol{Y_t} + \widetilde{\boldsymbol{r}}^{\boldsymbol{eq}\prime}\boldsymbol{\Sigma}\boldsymbol{\omega_{t+1}},
\end{aligned} \tag{B.36}$$

where $\widetilde{r}_{t+1}^{eq}$ is the log *nominal* return of equity from t to $t+1$, $\widetilde{\xi}_0^{eq}$ is constant, $\widetilde{\boldsymbol{\xi}}_{\mathbf{1}}^{\boldsymbol{eq}}$ is a vector of state vector coefficients, and $\widetilde{\boldsymbol{r}}^{\boldsymbol{eq}}$ is a vector of shock coefficients. Thus, this step involves linear approximation.

General expression To acknowledge the errors that are potentially caused by the linear approximations (the Taylor approximation in log price-dividend ratio in the return equation), we write down the return innovations for asset i with an idiosyncratic shock:

$$\widetilde{r}_{t+1}^{i} - E_t\left(\widetilde{r}_{t+1}^{i}\right) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}\boldsymbol{\omega_{t+1}} + \varepsilon_{t+1}^{i}, \tag{B.37}$$

where $E_t\left(\widetilde{r}_{t+1}^{i}\right)$ is the expected return, $\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}$ (10×1) is the asset i return loadings on selected state variable innovations (the choice of which depends on the asset classes), and ε_{t+1}^{i} is the Gaussian noise uncorrelated with the state variable shocks but may be cross-correlated (with other asset-specific shocks). The Gaussian shock ε_{t+1}^{i} has an unconditional variance σ_i^2.

B.4 Model-implied moments

In this section, we derive three model-implied asset conditional moments— expected excess returns, physical and risk-neutral conditional variances of nominal asset returns. The moments are crucial in creating the moment conditions during the third step of model estimation.

B.4.1 One-period expected excess return

We impose the no-arbitrage condition, $1 = E_t[\exp(\widetilde{m}_{t+1} + \widetilde{r}_{t+1}^{i})]$ ($\forall i \in$\{equity, treasury bond, corporate bond\}), and obtain the expected excess returns. Expand the law of one price (LOOP) equation:

$$\begin{aligned}
1 \quad &= \quad E_t[\exp(\widetilde{m}_{t+1} + \widetilde{r}_{t+1}^{i})] \\
&= \quad \exp\left[E_t(\widetilde{m}_{t+1}) + E_t(\widetilde{r}_{t+1}^{i})\right] \\
&\cdot \quad \exp\left\{\left[-\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1 - \sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right]p_t + \left[-\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1 - \sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right]n_t\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1 - \sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right]lp_t + \left[-\sigma_q(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1 - \sigma_q(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right]q_t\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1 - \sigma_{ln}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right]v_n\right\} \\
&\cdot \quad \exp\left\{\frac{1}{2}\left[(\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}_{\mathbf{1}}'(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \sigma_i^2\right]\right\},
\end{aligned} \tag{B.38}$$

where $\widetilde{\boldsymbol{m}}_{\mathbf{1}}$, $\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}$, σ_i, $\boldsymbol{S_1}$, and $\boldsymbol{\Sigma}^{\boldsymbol{other}}$ are constant matrices defined earlier, and

$$\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) \quad = \quad (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{1}},$$

$$\begin{aligned}
\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) &= (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{2}}, \\
\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) &= (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{4}}, \\
\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) &= (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{5}}, \\
\sigma_q(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) &= (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{9}}. \qquad \text{(B.39)}
\end{aligned}$$

Given the nominal risk free rate derived earlier using real pricing kernel and inflation, the nominal excess return is,

$$\begin{aligned}
E_t(\widetilde{r}^i_{t+1}) - \widetilde{rf}_t &= \left\{\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}})}\right]\right\} p_t \\
&+ \left\{\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}})}\right]\right\} n_t \\
&+ \left\{\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}})}\right]\right\} lp_t \\
&+ \left\{\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\mathbf{1}})}\right]\right\} q_t \\
&+ \left\{\sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \ln\left[\frac{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\mathbf{1}})}\right]\right\} v_n - \widetilde{\boldsymbol{m}}_{\mathbf{1}}'\boldsymbol{S}_{\mathbf{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}_{\mathbf{1}}'\widetilde{\boldsymbol{r}}^{\boldsymbol{i}} - \frac{1}{2}\left[\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{S}_{\mathbf{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}_{\mathbf{1}}'\widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2\right]
\end{aligned} \qquad \text{(B.40)}$$

where

$$\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\mathbf{1}}, \qquad \text{(B.41)}$$

$$\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\mathbf{2}}, \qquad \text{(B.42)}$$

$$\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\mathbf{4}}, \qquad \text{(B.43)}$$

$$\sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\mathbf{5}}, \qquad \text{(B.44)}$$

$$\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{\Sigma}_{\bullet\mathbf{9}}, \qquad \text{(B.45)}$$

$$\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{1}}, \qquad \text{(B.46)}$$

$$\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{2}}, \qquad \text{(B.47)}$$

$$\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{4}}, \qquad \text{(B.48)}$$

$$\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{5}}, \qquad \text{(B.49)}$$

$$\sigma_q(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) = (\widetilde{\boldsymbol{m}}_{\mathbf{1}}' + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\mathbf{9}}. \qquad \text{(B.50)}$$

B.4.2 One-period physical conditional return variance

The physical variance is easily obtained given the loadings:

$$\begin{aligned}
VAR_t(\widetilde{r}^i_{t+1}) &= \left(\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 p_t + \left(\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 n_t + \left(\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 lp_t + \left(\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 q_t \\
&+ \left(\sigma_{ln}\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)^2 v_n + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{S}_{\mathbf{1}}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}_{\mathbf{1}}'\widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2.
\end{aligned} \qquad \text{(B.51)}$$

B.4.3 One-period risk-neutral conditional return variance

To obtain the risk-neutral variance of the asset returns, we use the moment generating function under the risk-neutral measure:

$$\begin{aligned}
mgf_t^Q(\widetilde{r}^i_{t+1};\nu) &= \frac{E_t\left[\exp\left(\widetilde{m}_{t+1} + \nu\widetilde{r}^i_{t+1}\right)\right]}{E_t\left[\exp\left(\widetilde{m}_{t+1}\right)\right]} \\
&= \exp\left\{E_t(\widetilde{m}_{t+1}) + \nu E_t(\widetilde{r}^i_{t+1})\right\} \\
&\cdot \exp\left\{\left[-\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_p(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] p_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_n(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] n_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\mathbf{1}} + \nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] lp_t\right\}
\end{aligned}$$

$$
\begin{aligned}
&\cdot \quad \exp\left\{\left[-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] q_t\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})\right)\right] v_n\right\} \\
&\cdot \quad \exp\left\{\frac{1}{2}\left[(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) + \nu^2\sigma_i^2\right]\right\} \\
&/ \quad \exp\left\{E_t(\widetilde{m}_{t+1})\right\} \\
&/ \quad \exp\left\{\left[-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] p_t + \left[-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] n_t\right\} \\
&/ \quad \exp\left\{\left[-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] lp_t + \left[-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] q_t\right\} \\
&/ \quad \exp\left\{\left[-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}) - \ln\left(1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\right)\right] v_n + \frac{1}{2}\left[\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}\right]\right\} \\
&= \quad \exp\left\{\nu E_t(\widetilde{r}^i_{t+1})\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_p(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] p_t\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_n(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] n_t\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_{lp}(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] lp_t\right\} \\
&\cdot \quad \exp\left\{\left[-\sigma_q(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] q_t\right\} \\
&\cdot \quad A(\nu), \qquad \text{(B.52)}
\end{aligned}
$$

where

$$
\begin{aligned}
A(\nu) &= \exp\left\{\left[-\sigma_{ln}(\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \ln\left(\frac{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)\right] v_n\right\} \\
&+ \exp\left\{\frac{1}{2}\left[(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}) - \widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}\widetilde{\boldsymbol{m}}_{\boldsymbol{1}} + \nu^2\sigma_i^2\right]\right\} \qquad \text{(B.53)}
\end{aligned}
$$

, and

$$
\begin{aligned}
\sigma_p(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{1}}, \qquad \text{(B.54)} \\
\sigma_n(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{2}}, \qquad \text{(B.55)} \\
\sigma_{lp}(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{4}}, \qquad \text{(B.56)} \\
\sigma_{ln}(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{5}}, \qquad \text{(B.57)} \\
\sigma_q(\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}) &= (\widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}+\nu\widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime})\boldsymbol{\Sigma}_{\bullet\boldsymbol{9}}. \qquad \text{(B.58)}
\end{aligned}
$$

The first-order moment is the first-order derivate at $\nu = 0$:

$$
\begin{aligned}
E_t^Q(\widetilde{r}^i_{t+1}) &= \frac{\partial mgf_t^Q(\widetilde{r}^i_{t+1};\nu)}{\partial\nu}|_{\nu=0} \\
&= E_t(\widetilde{r}^i_{t+1}) + \frac{\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}p_t + \frac{\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}n_t + \frac{\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}lp_t + \frac{\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}q_t \\
&+ \frac{\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})\sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}v_n + \widetilde{\boldsymbol{m}}'_{\boldsymbol{1}}\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S}'_{\boldsymbol{1}}\widetilde{\boldsymbol{r}}^{\boldsymbol{i}}. \qquad \text{(B.59)}
\end{aligned}
$$

Note the similarity between $E_t(\widetilde{r}^i_{t+1}) - E_t^Q(\widetilde{r}^i_{t+1})$ from this equation and the equity premium derived before using the no-arbitrage condition. The second-order moment is derived,

$$
\begin{aligned}
VAR_t^Q(\widetilde{r}^i_{t+1}) &= E_t^Q\left((\widetilde{r}^i_{t+1})^2\right) - \left(E_t^Q(\widetilde{r}^i_{t+1})\right)^2 \\
&= \frac{\partial^2 mgf_t^Q(\widetilde{r}^i_{t+1};\nu)}{\partial\nu^2}|_{\nu=0} - \left(\frac{\partial mgf_t^Q(\widetilde{r}^i_{t+1};\nu)}{\partial\nu}|_{\nu=0}\right)^2 \\
&= \left(\frac{\sigma_p(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_p(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 p_t + \left(\frac{\sigma_n(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_n(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 n_t + \left(\frac{\sigma_{lp}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{lp}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 lp_t + \left(\frac{\sigma_q(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_q(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 q_t
\end{aligned}
$$

$$+ \quad \left(\frac{\sigma_{ln}(\widetilde{\boldsymbol{r}}^{\boldsymbol{i}})}{1-\sigma_{ln}(\widetilde{\boldsymbol{m}}_{\boldsymbol{1}})}\right)^2 v_n + \widetilde{\boldsymbol{r}}^{\boldsymbol{i}\prime}\boldsymbol{S_1}\boldsymbol{\Sigma}^{\boldsymbol{other}}\boldsymbol{S_1'}\widetilde{\boldsymbol{r}}^{\boldsymbol{i}} + \sigma_i^2. \tag{B.60}$$

C Variables and parameters

Table C.1: Variables. (In order of first appearance)

Symbol	
C_t	consumption level
Q_t	the relative risk aversion state (RRA) variable
m_t	log real pricing kernel
c_t	$\ln(C_t)$
q_t	$\ln(Q_t)$
Δc_t	log change in consumption
Δq_t	log change in RRA of per period utility of the representative agent
H_t	external habit level (as in Campbell and Cochrane, 1999)
θ_t	log change in the real industrial production index, or growth
p_t	upside macroeconomic uncertainty state variable, or "good" uncertainty, or shape parameter of the upside macroeconomic shock
n_t	downside macroeconomic uncertainty state variable, or "bad" uncertainty, or shape parameter of the downside macroeconomic shock
u_t^θ	growth disturbance
$\omega_{p,t}$	upside macroeconomic shock
$\omega_{n,t}$	downside macroeconomic shock
$\boldsymbol{Y_t^{mac}}$	macroeconomic state variables consisting of $\{\theta_t, p_t, n_t\}$
l_t	log corporate bond loss rate
u_t^l	loss rate-specific shock
$\omega_{lp,t}$	upside loss rate (cash flow) shock
$\omega_{ln,t}$	downside loss rate (cash flow) shock
lp_t	upside loss rate (cash flow) uncertainty state variable, or shape parameter of the upside loss rate shock
$\boldsymbol{Y_t^{fin}}$	financial state variables consisting of $\{l_t, lp_t\}$
g_t	change in log earnings
u_t^g	earnings growth-specific disturbance
$\omega_{g,t}$	standardized earnings growth-specific shock
κ_t	log consumption-earnings ratio
u_t^κ	consumption-earnings ratio-specific disturbance
$\omega_{\kappa,t}$	standardized consumption-earnings ratio-specific shock
η_t	log dividend payout ratio
u_t^η	dividend payout ratio-specific disturbance
$\omega_{\eta,t}$	standardized dividend payout ratio-specific shock
Δd_t	log change in dividend
u_t^q	risk aversion-specific disturbance
$\omega_{q,t}$	risk aversion shock
π_t	inflation
u_t^π	inflation-specific disturbance
$\omega_{\pi,t}$	standardized inflation-specific shock
Y_t^{other}	a vector of non-macro state variables, $[\pi_t, l_t, g_t, \kappa_t, \eta_t, lp_t, q_t]'$
Y_t	a vector of all 10 state variables, $[\boldsymbol{Y_t^{mac,\prime}}, \boldsymbol{Y_t^{fin,\prime}}]'$
ω_t	a vector of 9 independent shocks, $[\omega_{p,t}, \omega_{n,t}, \omega_{\pi,t}, \omega_{lp,t}, \omega_{ln,t}, \omega_{g,t}, \omega_{\kappa,t}, \omega_{\eta,t}, \omega_{q,t}]'$
$\widetilde{m}_t$	log nominal pricing kernel
$\widetilde{rf}_t$	nominal risk free rate
pc_t^1	log price-coupon ratio of one period defaultable bond portfolio
pc_t^N	log price-coupon ratio of N-period defaultable bond portfolio
PD_t	price-dividend ratio
$\tilde{r}_t^i$	log nominal asset return for asset i, $i \in \{eq, cb\}$
$E_t\left(r_{t+1}^i\right)$	expected return for asset i
RP_t^i	model-implied one-month expected excess returns for asset i

$VAR_t^i \equiv VAR_t(\tilde{r}_{t+1}^i)$	model-implied one-month expected physical variances for asset i
$VAR_t^{i,Q} \equiv VAR_t^Q(\tilde{r}_{t+1}^i)$	model-implied one-month expected risk-neutral variances for asset i
$RVAR_t^i$	empirical benchmark of one-month realized physical variances for asset i
$QVAR_t^{eq}$	empirical benchmark of one-month expected risk-neutral variances for equity
E_t	monthly earnings
ra_t^{BEX}	Bekaert-Engstrom-Xu's financial proxy to risk aversion
unc_t^{BEX}	Bekaert-Engstrom-Xu's financial proxy to macroeconomic uncertainty

Table C.2: Parameters.

Symbol	
γ	constant utility curvature parameter
a, b	parameters in Q_t
β	constant discount factor
$\bar{\theta}$	unconditional mean of growth
ρ_θ	AR(1) coefficient of growth
m_p	sensitivity of output growth on lagged upside macroeconomic uncertainty
m_n	sensitivity of output growth on lagged downside macroeconomic uncertainty
$\bar{p}$	unconditional mean of p_t
$\bar{n}$	unconditional mean of n_t
$\sigma_{\theta p}$	scale parameter associated with the upside macroeconomic shock, $\omega_{p,t}$, in θ_t
$\sigma_{\theta n}$	scale parameter associated with the downside macroeconomic shock, $\omega_{n,t}$, in θ_t
ρ_p	AR(1) coefficient of p_t
ρ_n	AR(1) coefficient of n_t
σ_{pp}	sensitivity of p_t on $\omega_{p,t}$
σ_{nn}	sensitivity of n_t on $\omega_{n,t}$
l_0	constant in the dynamic process of l_t
ρ_{ll}	AR(1) coefficient of l_t
m_{lp}	sensitivity of loss rate on lagged upside macroeconomic uncertainty
m_{ln}	sensitivity of loss rate on lagged downside macroeconomic uncertainty
σ_{lp}	sensitivity of loss rate on the upside macroeconomic shock $\omega_{p,t}$
σ_{ln}	sensitivity of loss rate on the downside macroeconomic shock $\omega_{n,t}$
σ_{llp}	scale parameter associated with $\omega_{lp,t}$ in l_t
σ_{lln}	scale parameter associated with $\omega_{ln,t}$ in l_t
$\bar{lp}$	unconditional mean of lp_t
$\bar{ln}$	constant shape parameter of $\omega_{ln,t}$
σ_{lplp}	sensitivity of lp_t on $\omega_{lp,t}$
ρ_{lp}	AR(1) coefficient of lp_t
j_0 *	constant in the dynamic process of variable j_t
ρ_{jj} *	AR(1) coefficient of variable j_t
$\boldsymbol{\rho_{j,mac}}$ *	sensitivities of variable j_t on lagged macroeconomic state variables $\boldsymbol{Y_t^{mac}}$
$\boldsymbol{\rho_{j,fin}}$ **	sensitivities of variable j_t on lagged financial state variables $\boldsymbol{Y_t^{fin}}$
σ_{jp}*	sensitivity of variable j_t on the upside macroeconomic shock $\omega_{p,t}$
σ_{jn} *	sensitivity of variable j_t on the downside macroeconomic shock $\omega_{n,t}$
σ_{jlp}**	sensitivity of variable j_t on the upside loss rate shock $\omega_{lp,t}$
σ_{jln} **	sensitivity of variable j_t on the downside loss rate shock $\omega_{ln,t}$
σ_{jj} *	standard deviation of the variable j_t residual
q_0	constant in the dynamic process of risk aversion q_t
ρ_{qq}	AR(1) coefficient of q_t
ρ_{qp}	sensitivity of risk aversion on lagged upside macroeconomic shock
ρ_{qn}	sensitivity of risk aversion on lagged downside macroeconomic shock
σ_{qp}	sensitivity of risk aversion on the upside macroeconomic shock $\omega_{p,t}$
σ_{qn}	sensitivity of risk aversion on the downside macroeconomic shock $\omega_{n,t}$
σ_{qq}	scale parameter associated with the risk aversion shock $\omega_{q,t}$
$\boldsymbol{\mu}$	constant vector in the state variable system (10 × 1)
$\boldsymbol{A}$	autocorrelation vector in the state variable system (10 × 10)
$\boldsymbol{\Sigma}$	scale / volatility parameter matrix of the 9 shocks (10 × 9)
m_0	constant in the real pricing kernel process
$\boldsymbol{m_1}$	sensitivity vector of real pricing kernel to state variable shocks

$\boldsymbol{m_2}$	sensitivity vector of real pricing kernel to state variable levels
$\tilde{m}_0$	constant in the nominal pricing kernel process
$\widetilde{\boldsymbol{m}}_{\boldsymbol{1}}$	sensitivity vector of nominal pricing kernel to state variable shocks
$\widetilde{\boldsymbol{m}}_{\boldsymbol{2}}$	sensitivity vector of nominal pricing kernel to state variable levels
b_0^1	constant in the log price-coupon ratio of one period defaultable bond portfolio
$\boldsymbol{b_1^1}$	sensitivity vector of the log price-coupon ratio on state variable levels
b_0^N	constant in the log price-coupon ratio of N-period defaultable bond portfolio
$\boldsymbol{b_1^N}$	sensitivity vector of the log price-coupon ratio on state variable levels
$\tilde{\xi}_0^i$	constant in the log return generating process of asset i
$\tilde{\boldsymbol{\xi}}_{\boldsymbol{1}}^{\boldsymbol{i}}$	sensitivity vector of log asset return i on lagged state variable levels
$\tilde{\boldsymbol{r}}^{\boldsymbol{i}}$	asset return i loadings on state variable shocks
σ_i	unconditional volatility of idiosyncratic return residuals
$\boldsymbol{\chi}$	q_t loadings on the instruments
$\boldsymbol{\chi}^{\boldsymbol{unc}}$	macroeconomic uncertainty loadings on the instruments
$\boldsymbol{\Theta}$	a vector of unknown parameters in the GMM system
$\widehat{X}$	estimates of X where X can be a parameter or a variable

* for all $j \in \{g, \kappa, \eta, \pi\}$:
** for all $j \in \{g, \kappa, \eta\}$:

D Supplementary Tables and Figures

Table D.1: Summary Statistics of Financial Instruments Spanning Risk Aversion

This table presents summary statistics of the 6 financial instruments that are used to span our risk aversion measure: "tsprd" is the difference between 10-year treasury yield and 3-month Treasury yield; "csprd" is the difference between Moody's Baa yield and the 10-year zero-coupon Treasury yield; "EY5yr" ("DY5yr") is the detrended earnings (dividend) yield where the moving average takes the 5 year average of monthly earnings yield, starting one year before; "rvareq" and "rvarcb" are realized variances of log equity returns and log corporate bond returns, calculated from daily returns; "qvareq" is the risk-neutral conditional variance of log equity returns; for the early years (before 1990), we use VXO and authors' calculations. Bold (italic) coefficients have <5% (10%) p-values. Block bootstrapped errors are shown in parentheses. The sample period is from 1986/06 to 2015/02 (345 months).

	tsprd	csprd	DY5yr	EY5yr	rvareq	qvareq	rvarcb
			Correlation Matrix				
tsprd	1	**0.3524**	**0.2595**	**0.2526**	**0.1269**	**0.1244**	**0.2952**
csprd		1	**0.4990**	**0.5083**	**0.4786**	**0.5988**	**0.5330**
DY5yr			**1.0000**	**0.8966**	**0.1678**	**0.1650**	**0.3101**
EY5yr				1	**0.1399**	**0.1564**	**0.3359**
rvareq					1	**0.8431**	**0.5943**
qvareq						1	**0.5376**
rvarcb							1
			Summary Statistics				
Mean	**0.0179**	**0.0231**	**-0.0030**	**-0.0074**	**0.0029**	**0.0040**	**0.0002**
Boot.SE	(0.0006)	(0.0004)	(0.0003)	(0.0008)	(0.0003)	(0.0002)	(0.0000)
S.D.	**0.0116**	**0.0075**	**0.0061**	**0.0149**	**0.0059**	**0.0037**	**0.0003**
Boot.SE	(0.0003)	(0.0005)	(0.0003)	(0.0007)	(0.0014)	(0.0005)	(0.0000)
Skewness	**-0.2322**	**1.7891**	0.0959	**-0.3495**	**8.1198**	**3.7225**	**4.2227**
Boot.SE	(0.0810)	(0.2515)	(0.1882)	(0.1502)	(1.5951)	(0.5123)	(0.6872)
AR(1)	**0.9668**	**0.9640**	**0.9822**	**0.9843**	**0.4312**	**0.7462**	**0.5775**
SE	(0.0137)	(0.0143)	(0.0083)	(0.0068)	(0.0488)	(0.0360)	(0.0441)

Table D.2: Projecting Pure Cash Flow Uncertainty using OLS

This table presents regression results of the estimated monthly pure cash flow uncertainty (from loss rate) on a set of monthly asset prices; some are used to span the time-varying risk aversion. The dependent variable is lp_t, the time-varying shape parameter of the pure right-tail loss rate residual (after controlling for macroeconomic shocks) as demonstrated in Table 2. $\times 10^{-3}$" in the header means that the coefficients and their SEs reported are divided by 1000 for reporting convenience. "VARC" reports the variance decomposition. Bold (italic) coefficients have <5% (10%) p-values. Robust and efficient standard errors are shown in parentheses. Adjusted R^2s are reported. The sample period is 1986/06 to 2015/02 (345 months).

	($\times 10^{-3}$)	
	lp_t	VARC
constant	*-0.001*	
	(0.001)	
χ_{tsprd}	**-0.058**	-2.33%
	(0.011)	
χ_{csprd}	**0.202**	62.69%
	(0.025)	
χ_{DY5yr}	**0.234**	41.57%
	(0.046)	
χ_{EY5yr}	**-0.061**	-22.57%
	(0.019)	
χ_{rvareq}	-0.026	-3.76%
	(0.062)	
χ_{qvareq}	*0.119*	13.25%
	(0.067)	
χ_{rvarcb}	**1.779**	13.67%
	(0.593)	
$\chi_{rvarcbSPEC}$	-0.223	-2.51%
	(0.556)	
R^2	9.11%	

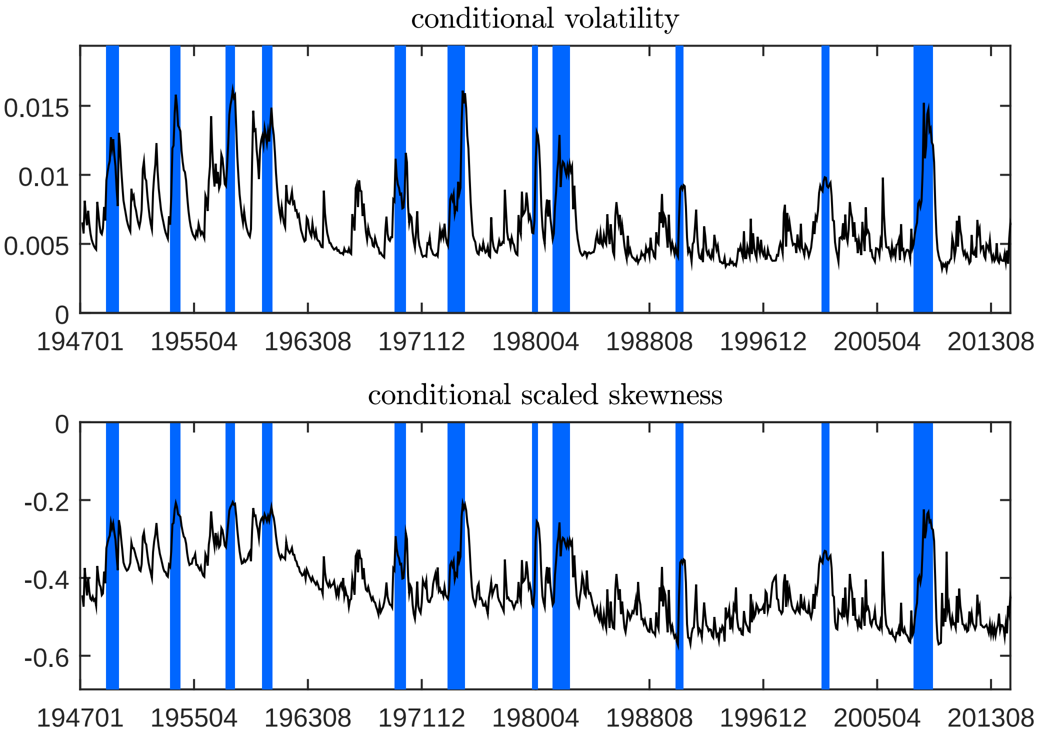

Figure D.1: Model-implied conditional moments of industrial production growth. The shaded regions are NBER recession months from the NBER website.

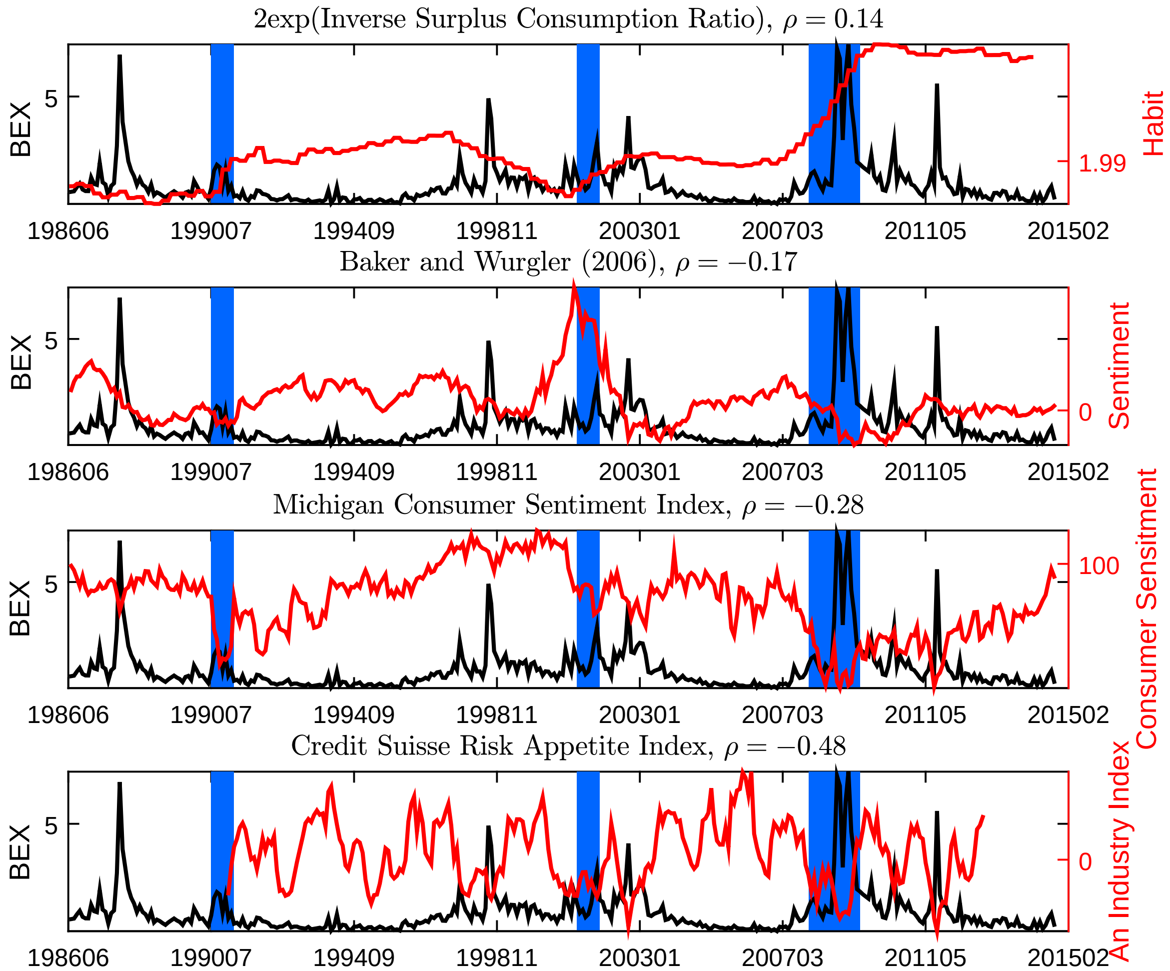

Figure D.2: Risk aversion/sentiment measures: A comparison.

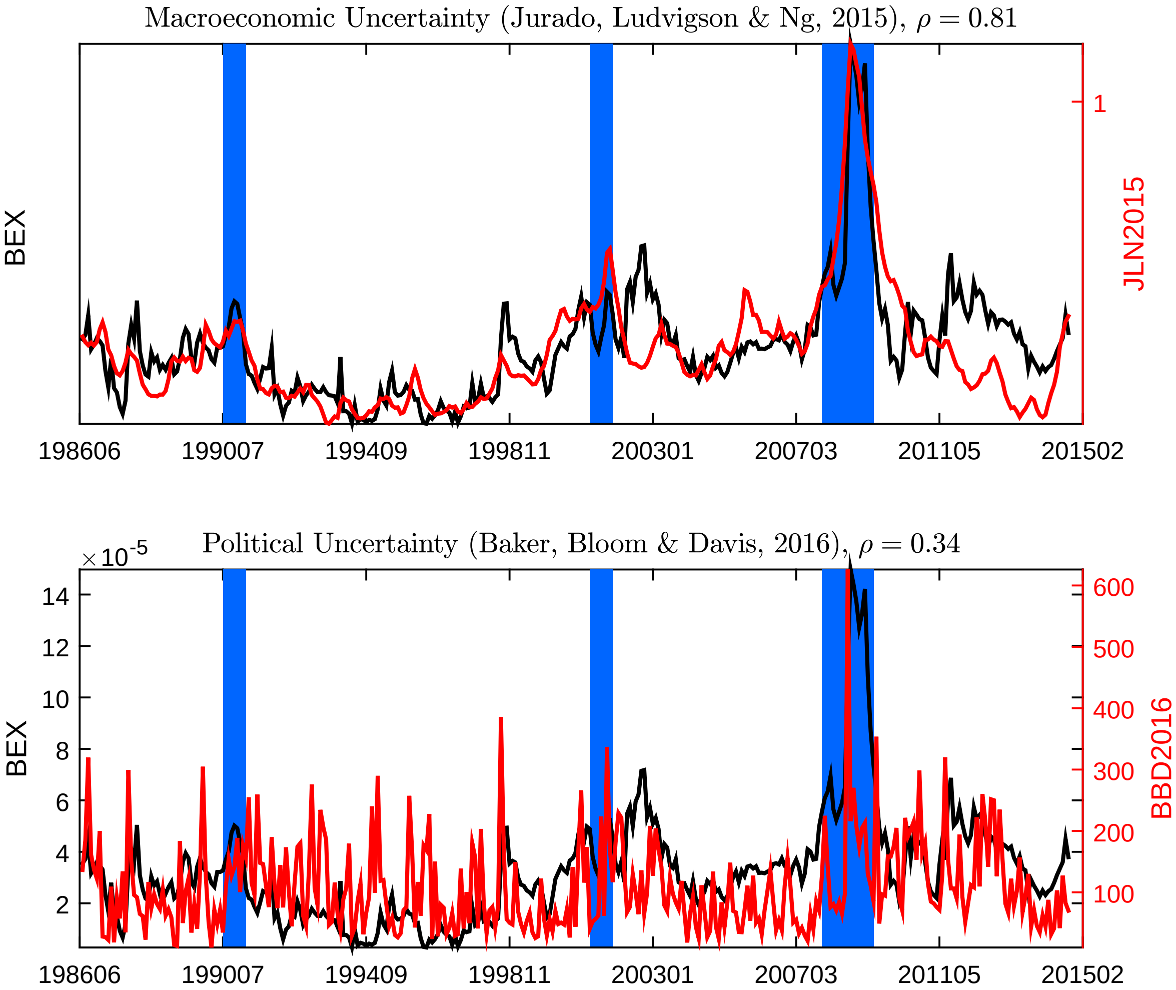

Figure D.3: Uncertainty measures: A comparison.