# "Idiosyncratic Volatility and the Intertemporal Capital Asset Pricing Model"

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Discussion by:

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# Motivation: Idiosyncratic volatility

- A long literature on volatility, idiosyncratic volatility, and how they predict stock returns (market and cross section): French, Schwert & Stambaugh (1987); Goyal & Santa-Clara (2003); Bali et al. (2005); Wei & Zhang (2005); Malkiel & Xu (2002); Ang et al. (2006); Stambaugh et al. (2015); and so on

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- **An ongoing debate: Average idiosyncratic volatility seems to be able to predict future returns, but evidence is mixed given equal- or value-weighting scheme of idiosyncratic volatilities (e.g., Goyal & Santa-Clara (2003); Guo & Savickas (2008))**

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- **Key insight of this paper: Aggregate idiosyncratic volatilities ($IV$) should be related to conditional covariance risk ($\sigma_{MH}$), hence predicting returns**
  - ⇒ Analytical derivation
  - ⇒ Empirical proxies for risk exposures to the unobserved hedge portfolio
  - ⇒ Implications

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- The idiosyncratic variance under a mis-specified model contains useful information about the covariance risk and therefore matters for asset prices $\Rightarrow$

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- Suppose a (misspecified) single-index model:
  $R_{i,t+1} = b_{iM,t} R_{M,t+1} + \eta_{i,t+1}$
- Individual idiosyncratic variance: $Var_t(\eta_{i,t+1}) = A_{iH,t}\sigma^2_{H,t} - A_{iM,t}\sigma_{MH,t}$

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- Conditional covariance between market and hedge portfolio:

$$\sigma_{MH,t} = \frac{A_t^S}{A_t^F B_t^S - A_t^S B_t^F}\widetilde{IV}_t^F - \frac{A_t^F}{A_t^F B_t^S - A_t^S B_t^F}\widetilde{IV}_t^S,$$

$\Rightarrow$ Conditional excess returns:

$$\mu_{i,t} = \beta_{iM,t} \times \mu_{M,t} + C_{i,t}^F \times \widetilde{IV}_t^F - C_{i,t}^S \times \widetilde{IV}_t^S,$$

$\Rightarrow$ Analytically, highlighted components $\propto$ conditional beta to Hedge portfolio excess returns $\beta_{iH,t}$ (risk exposure)

# What does this paper do? (2) Empirical results

- Three ways to proxy for $\beta_{iH,t}$:

$$\begin{cases} \beta_{iH,t} = C_{i,t}^{F} \times \frac{A_t^F B_t^S - A_t^S B_t^F}{\gamma_M A_t^S + \gamma_H B_t^F} \propto C_{i,t}^{F} \textit{ in cross-section} \\ \beta_{iH,t} = C_{i,t}^{S} \times \frac{A_t^F B_t^S - A_t^S B_t^F}{\gamma_M A_t^F + \gamma_H B_t^S} \propto C_{i,t}^{S} \textit{ in cross-section} \\ \beta_{iH,t} = \frac{(A_t^F B_t^S - A_t^S B_t^F)}{\gamma_H^2} \left(C_t^S \times C_{i,t}^F - C_t^F \times C_{i,t}^S\right) \propto \left(C_t^S \times C_{i,t}^F - C_t^F \times C_{i,t}^S\right) \textit{ in cross-section} \end{cases}$$

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- Result (1): Estimate $C_{i,t}^F$, $C_t^F$, $C_{i,t}^S$, $C_t^S$ through bivariate predictive regressions of market excess returns, and individual excess stock returns
  - ⋆ Novel method of proxying for risk exposure to hedge portfolio
  - ⋆ Coefficients of IVs seem robust, after various controls

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- Result (1): Estimate $C^F_{i,t}$, $C^F_t$, $C^S_{i,t}$, $C^S_t$ through bivariate predictive regressions of market excess returns, and individual excess stock returns
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  - ⋆ Coefficients of IVs seem robust, after various controls
- Result (2): Cross-section implications by sorting estimated $\beta_{iH,t}$
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- Result (3): Compute proxy for conditional covariance risk, $\sigma_{MH}$
  - ⋆ Highly correlated with Kelly-Jiang tail index at 0.7

# What I like about this paper:

1. Explore a novel way of estimating risk exposure to the latent hedge portfolio *without* estimating the conditional moments of hedge portfolio (variance and market covariance)
   - Cross-section of individual betas
   - Time-series property of conditional covariance risk
2. The writing is very clear and easy to follow

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Table 2 Univariate and Bivariate Predictive Time-series Regressions

| Panel A. Univariate Regression | | | | | | |
|---|---|---|---|---|---|---|
| **192801 to 201912** | **Coefficient** | **K=1** | **K=3** | **K=6** | **K=12** | **K=24** |
| $IV_t^F$ | b | 0.128 | 0.075 | 0.021 | 0.075 | 0.055 |
| | t-stat | 0.36 | 0.24 | 0.10 | 0.34 | 0.33 |
| | adj. $R^2$ (%) | -0.03 | -0.04 | -0.08 | 0.11 | 0.14 |
| $IV_t^S$ | b | -0.010 | -0.079 | -0.124 | -0.066 | -0.065 |
| | t-stat | -0.03 | -0.28 | -0.63 | -0.29 | -0.35 |
| | adj. $R^2$ (%) | -0.09 | -0.03 | 0.20 | 0.07 | 0.23 |
| **Panel B. Bivariate Regression** | | | | | | |
| **192801 to 201912** | **Coefficient** | **K=1** | **K=3** | **K=6** | **K=12** | **K=24** |
| $IV_t^F$ | b | 1.561 | 1.710 | 1.596 | 1.573 | 1.319 |
| | (t-stat) | (2.14) | (2.34) | (2.75) | (3.43) | (4.74) |
| $IV_t^S$ | b | -1.501 | -1.714 | -1.650 | -1.573 | -1.329 |
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  - ⋆ This explains why we can get similar $\beta_{iH,t} \propto C_{i,t}^F$ and $\beta_{iH,t} \propto C_{i,t}^S$, given the cross-section results.

# #2: Result generalization

- The three expressions for $\beta_{iH}$ are key to the paper, and should be examined and shown to deliver stable results. I understand VW and EW is an interesting ongoing debate; but **this methodology is not married to the two weighting schemes** only. In addition, considering other (less obviously correlated) ones **may help resolve the collinearity concern** as well:

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- How important is the non-$C_{i,t}$ part of $\beta_{i,t}$?

# #3: Other comments

- The key innovation is to derive and show that we need two weighting schemes for this method to work. To be able to obtain analytical unique solution for the two unknowns, (a) conditional variance of the hedge portfolio, and (b) its conditional convariance risk

  ⋆ Suggestion: Consider moving this equation from Appendix to the main text

$$\begin{cases} \widetilde{IV}_t^F \equiv IV_t^F - \Omega_t^F = A_t^F \sigma_{H,t}^2 - B_t^F \sigma_{MH,t} \\ \widetilde{IV}_t^S \equiv IV_t^S - \Omega_t^S = A_t^S \sigma_{H,t}^2 - B_t^S \sigma_{MH,t} \end{cases} . \qquad \text{(A. 11)}$$

- Typo in Equation (10)? Second beta should be $\beta_{MH,t}$

- Pitch / first paragraph. The weighting choice, as readers would later realize, becomes less of the focus of the paper; instead, I think the novel beta estimates (without estimating hedge portfolio moments) is very cool!

# Conclusion

- **I highly recommend this paper!**
- Potential places to improve:
  1. Address model stability — Comments #1 & #2
  2. Packaging suggestions — Comments #3

## Thank You!

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