Procyclicality of the Comovement between Dividend Growth and Consumption Growth

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Abstract

Duffee (2005) documents that the conditional covariance between market returns and consumption growth, or the amount of consumption risk, is procyclical. In light of this "Duffee Puzzle", I demonstrate empirically that the conditional covariance between the immediate cash flow part of market returns (dividend growth) and consumption growth is (1) procyclical and (2) a consistent source of procyclicality in the puzzle. Moreover, I solve an external habit formation model that incorporates realistic joint dynamics of consumption growth and dividend growth. The procyclical consumption-dividend comovement leads to two new procyclical terms in the amount of consumption risk via cash flows and valuation channels, respectively. These two procyclical terms play an important role in generating realistic magnitude of consumption risk, thus potentially accommodating the Duffee Puzzle.

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1 Introduction

Duffee (2005) documents that the amount of consumption risk (i.e., the conditional covariance between market returns and consumption growth) is procyclical. This finding makes it harder to explain countercyclical equity premium. In fact, many well-accepted consumption-based theories imply a countercyclical amount of consumption risk (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004; and their recent variants). Hence, I term this finding "the Duffee Puzzle".

In this paper, I first empirically confirm Duffee's (2005) main findings with an addition of 13 years of data to his sample. I then find that the conditional covariance between the immediate cash flow part of market returns (dividend growth) and consumption growth is procyclical and a consistent source of procyclicality in the puzzle. Moreover, I propose a parsimonious and new data generating process (DGP) for the joint dynamics of dividend and consumption growth featuring the procyclical comovement, and explore how it affects the performance of an external habit formation model.

In the first part of the paper, I use a dynamic conditional correlation model to identify the cyclicalities of the amount of risk and its two conditional covariance components given the return decomposition:

$$Cov_t\left(r^m_{t+1}, \Delta c_{t+1}\right) = \underbrace{Cov_t\left(\Delta d_{t+1}, \Delta c_{t+1}\right)}_{\text{Immediate Cash Flow}} + \underbrace{Cov_t\left(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1}\right)}_{\text{Valuation}}, \tag{1}$$

where r^m_{t+1} is the log market return, Δc_{t+1} is the log consumption growth, and Δd_{t+1} is the log dividend growth. I show that the return covariance behaves weakly procyclically, despite the strongly countercyclical market return and consumption growth volatilities during the sample period that now includes the 2007-08 financial crisis. I then show that the immediate cash flow covariance behaves *procyclically* while the valuation covariance behaves *countercyclically*. The former covariance is the only source of procyclicality in the amount of consumption risk given the decomposition, and thus potentially solves the puzzle. This is my core empirical finding.

To further illustrate my core finding, I examine the dynamics of the proportion of the total return covariance explained by immediate cash flow covariance. I find that this proportion varies greatly over time and comoves positively with the business cycle. It reaches a peak at 50% during the 1960s and late 1980s expansions, while it reaches a trough at -50% at the beginning of the 2007-08 financial crisis. Meanwhile, the unconditional proportion is 8% using the full sample (January 1959–June 2014). The empirical part of my paper concludes with a list of eight stylized facts pertinent to the Duffee Puzzle and its components. Six are new to the literature. These include the new facts that consumption-dividend correlation, covariance, beta (three comovement statistics), and dividend growth volatility are all procyclical.

In the remainder of this paper, I explore how incorporating realistic joint dynamics of consumption growth and dividend growth into an endowment economy affects equilibrium stock price dynamics, thus potentially accommodating the Duffee Puzzle. First, I formulate a new

DGP that matches both procyclical consumption-dividend comovement and countercyclical consumption growth volatility, which extant DGPs fail to do (e.g., Campbell and Cochrane, 1999; Bansal, Kiku, and Yaron, 2012; Segal, Shaliastovich, and Yaron, 2015; Bekaert and Engstrom, 2017). In my DGP, consumption receives a "fundamental" shock and an "event" shock per period. Fundamental shocks to consumption drive contemporaneous shocks to dividends. The dividend shocks react more to fundamental shocks in booms (times when past consumption growth has been high). This mechanism generates both procyclical dividend growth volatility and a procyclical conditional covariance between consumption and dividends. In addition, the DGP exhibits time-varying volatility of negatively skewed event shocks to consumption. This mechanism generates countercyclical consumption growth volatility. The estimation results of the DGP further reveal that fundamental shocks on average explain 82% of the total consumption growth variability, while event shocks account for 34%–58% of the total consumption growth variability during recessions.

Finally, I solve a variant of the Campbell and Cochrane model (henceforth, CC) that accommodates the new DGP. An approximate analytical solution suggests that the procyclical consumption-dividend comovement in the new DGP induces two new *procyclical* terms in the amount of consumption risk: dividend risk (via cash flows) and comovement risk (via valuation). The first term is introduced by the DGP. The second term captures how pricing is affected by a persistent and procyclical dividend variance induced by dividends' procyclical exposure to the fundamental consumption shock. When a positive fundamental shock occurs in this period, future dividends are expected to react more to future consumption shocks, driving up the expected dividend growth variance. This variance becomes capitalized in stock prices, leading stock prices to react more positively to consumption shocks during booms than during recessions. The numerical solution further shows that, without procyclical consumption-dividend comovement, a CC model with countercyclical risk aversion and macroeconomic uncertainty tends to generate an amount of consumption risk that is too high. My model generates more realistic magnitudes of consumption risk during both recession and non-recession periods.

The equity premium in my model can be expressed as the product of a countercyclical price of risk (which is consistent with CC) and a time-varying amount of risk that comprises both procyclical (which is new) and countercyclical terms. These terms reveal countervailing effects on the magnitude of both conditional and unconditional equity premiums. The model implies a conditional equity premium that no longer increases monotonically when consumption drops. This is because a negative fundamental consumption shock results in both higher risk aversion and lower procyclical terms in the amount of risk. In addition, the model generates a lower unconditional equity premium. This is because stock prices here incorporate procyclical risks, rendering the market asset less risky. The numerical solution confirms this theoretical prediction, which also yields a more realistic Sharpe Ratio.

It is noteworthy that Duffee (2005) relates this procyclical amount of consumption risk to a "composition effect": The agent's consumption growth is more correlated with stock returns when financial wealth and asset prices are high. Although my model does not explicitly involve modeling wealth and other capital, it does rationalize a positive relationship between procyclical consumption-dividend comovement and equity valuation ratios in equilibrium. The estimation results of the DGP show a significant and negative correlation between consumption-dividend comovement and the well-known *cay* variable from Lettau and Ludvigson (2001). Therefore, both explanations are potentially consistent.

The outline of this paper is as follows. Section 2 replicates the main empirical finding in Duffee (2005) and examines the cyclicalities of the immediate cash flow and valuation covariances. Section 3 formulates and estimates the new DGP. Section 4 analyzes a variant of Campbell and Cochrane's habit formation model that potentially accommodates the Duffee Puzzle. Concluding comments are offered in Section 5.

2 The Duffee Puzzle Revisited, Econometrically

The decomposition of the amount of consumption risk, as shown in Equation (1), yields an immediate cash flow conditional covariance, $Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$, and a valuation conditional covariance, $Cov_t\left(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1}\right)$. In this section, I exploit a bivariate dynamic dependence model in the GARCH class to replicate the Duffee Puzzle and identify the cyclicalities of the two conditional comovements that constitute the puzzle.

2.1 The Dynamic Dependence Model

The empirical analysis uses four variables: log consumption growth Δc_{t+1}, log market returns r^m_{t+1}, log dividend growth Δd_{t+1}, and their difference, $r^m_{t+1} - \Delta d_{t+1}$. I first project each series onto an exogenous business cycle indicator (1=recession, 0=non-recession) to obtain the series residuals denoted by $\widetilde{\epsilon}$. Consider a bivariate system,

$$\boldsymbol{\widetilde{\epsilon}_{t+1}} \equiv \begin{bmatrix} \widetilde{\epsilon}_{1,t+1} & \widetilde{\epsilon}_{2,t+1} \end{bmatrix}', \tag{2}$$

where, in this paper, $\widetilde{\epsilon}_{1,t+1}$ is the consumption growth residual and $\widetilde{\epsilon}_{2,t+1}$ could be the market return residual, the dividend growth residual, or the valuation residual.

I follow Engle (2002) and express the conditional variance-covariance matrix of the residuals, $\boldsymbol{H_t} \equiv E_t\left[\boldsymbol{\widetilde{\epsilon}_{t+1}\widetilde{\epsilon}'_{t+1}}\right]$, in a quadratic form,

$$\boldsymbol{H_t} = \boldsymbol{\Lambda_t Corr_t \Lambda_t}. \tag{3}$$

The diagonal terms of $\boldsymbol{\Lambda_t}$ (2×2) are the square roots of the conditional variances of $\widetilde{\epsilon}_{1,t+1}$ and $\widetilde{\epsilon}_{2,t+1}$, and the off-diagonal terms of $\boldsymbol{\Lambda_t}$ are zeros; $\boldsymbol{Corr_t}$ (2×2) is the conditional correlation matrix. Under quasi likelihood assumptions (Bollerslev and Wooldridge, 1992; White, 1996), the log-likelihood of this dynamic dependence model can be written as the sum of a volatility term and a correlation term. As a result, the model can be estimated by maximizing each term separately.

2.1.1 Conditional Variance

The empirical literature contains robust evidence that innovations of consumption growth and market returns are heteroskedastic (e.g., Bollerslev, Engle, and Wooldridge, 1988; Kandel and Stambaugh, 1990; Lettau, Ludvidgon, and Wachter, 2008; among many others) and non-Gaussian (e.g., Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993; Bekaert and Engstrom, 2017; among many others). However, there is scant research on the behavior of dividend growth conditional variance. Therefore, to capture realistic dynamics of condition variances, I consider several representative GARCH-class models for each residual series.

Denote h_t as the conditional variance at the information set t. The first model assumes that the residuals follow a conditional Gaussian distribution, $\widetilde{\epsilon}_{t+1} \sim N(0, h_t)$, and the conditional variance h_t is a two-state process,

$$h_t = \overline{h}\left(1 + q_t\right), \tag{4}$$

$$q_t = \nu SNBER_t, \tag{5}$$

where $\overline{h}$ denotes the predetermined unconditional variance; the process of q_t is a multiple of the standardized NBER recession indicator, denoted as $SNBER_t$, so that the average conditional variance $E(h_t)$ is $\overline{h}$; and ν is an unknown parameter. The zero-mean cyclical component q_t identifies the cyclicality within the model: a positive (negative) coefficient estimate of ν indicates a countercyclical (procyclical) conditional variance; a zero estimate fails to reject the null of an acyclical variance.[1]

The second and third conditional variance models introduce this cyclical component into a generalized autoregressive conditional heteroskedastic process as its long-run mean "$\overline{h}\left(1 + q_t\right)$":

$$h_t = \overline{h}\left(1 + q_t\right) + \alpha\left[\widetilde{\epsilon}_t^2 - \overline{h}\left(1 + q_{t-1}\right)\right] + \beta\left[h_{t-1} - \overline{h}\left(1 + q_{t-1}\right)\right], \tag{6}$$

where $\alpha + \beta < 1$, $\alpha > 0$, $\beta > 0$; q_t is modeled as in Equation (5). The second model assumes a conditional Gaussian distribution, while the third model assumes a symmetric leptokurtic conditional generalized error distribution (GED). The GARCH model (Bollerslev, 1987) and the GED-GARCH model (Nelson, 1991) are their special cases with $\nu = 0$, respectively.

The fourth model allows for asymmetries in both conditional variance and distributional assumptions. I use the "Bad Environment-Good Environment" (BEGE) model from Bekaert, Engstrom, and Ermolov (2015). The residual follows a composite distribution of two centered gamma shocks that independently govern left- and right-tail behaviors. For simplicity, I assume that most heteroskedasticity comes from the left tail. The mathematical expression of the composite residual is $\widetilde{\epsilon}_{t+1} = \sigma_{cp}\widetilde{\omega}_{cp,t+1} - \sigma_{cn}\widetilde{\omega}_{cn,t+1}$, where $\widetilde{\omega}_{cp,t+1} \sim \widetilde{\Gamma}(\overline{cp}, 1)$, $\widetilde{\omega}_{cn,t+1} \sim \widetilde{\Gamma}(cn_t, 1)$, shape parameters $\overline{cp} > 0$ and $cn_t > 0$, and scale parameters $\sigma_{cp} > 0$ and $\sigma_{cn} > 0$. Given the distributional assumption, the total conditional variance is $h_t = \sigma_{cp}^2\overline{cp} + \sigma_{cn}^2 cn_t$. The left-tail shape parameter cn_t has isomorphic GARCH-class dynamics with a cyclical long-run mean

[1] This instrument approach is popular in empirical studies (e.g., Campbell, 1987; Shanken, 1990; Bekaert and Harvey, 1995; Duffee, 2005; among many others).

"$\overline{cn}\,(1+q_t)$":

$$cn_t = \overline{cn}\,(1+q_t) + \alpha_{cn}\left[\frac{\widetilde{\epsilon}_t^2}{2\sigma_{cn}^2} - \overline{cn}\,(1+q_{t-1})\right] + \beta_{cn}\left[cn_{t-1} - \overline{cn}\,(1+q_{t-1})\right], \tag{7}$$

where $\alpha_{cn}+\beta_{cn}<1$, $\alpha_{cn}>0$, $\beta_{cn}>0$; $\overline{cn}$ is now an unknown parameter because it depends on the value of σ_{cn}; q_t is modeled similarly.

2.1.2 Conditional Correlation

Denote two standardized residuals by $\boldsymbol{z_{t+1}} \equiv \begin{bmatrix} z_{1,t+1} & z_{2,t+1} \end{bmatrix}' = \boldsymbol{\Lambda_t^{-1}}\widetilde{\boldsymbol{\epsilon}}_{\boldsymbol{t+1}}$. To simultaneously test the cyclicality of time-varying correlation, my model builds a cyclical long-run component into the dynamic conditional correlation process:

$$\begin{aligned}\boldsymbol{Q_t} = \overline{Q}_{12}\begin{bmatrix} 1 & 1+q_t \\ 1+q_t & 1 \end{bmatrix} &+ \alpha_{12}\left[\boldsymbol{z_t z_t'} - \overline{Q}_{12}\begin{bmatrix} 1 & 1+q_{t-1} \\ 1+q_{t-1} & 1 \end{bmatrix}\right] \\ &+ \beta_{12}\left[\boldsymbol{Q_{t-1}} - \overline{Q}_{12}\begin{bmatrix} 1 & 1+q_{t-1} \\ 1+q_{t-1} & 1 \end{bmatrix}\right], \end{aligned} \tag{8}$$

where the parameter $\overline{Q}_{12}$ denotes the predetermined constant correlation of the standardized residuals. As introduced before, $q_t = \nu SNBER_t$. I call it the "DCC-q_t" model in this paper. It is noteworthy that Engle (2002) assumes a constant long-run mean, or a "$\nu = 0$" special case. Meanwhile, Colacito, Engle, and Ghysels (2011) use a weighted average of past correlations to model the long-run conditional mean. Unlike these DCC models, this DCC-q_t model aims to link time-varying correlation to the business cycle through the new cyclical long-run component.[2]

2.2 Data

I follow Duffee (2005) to use monthly data indexed with t. Monthly real consumption is defined as the sum of seasonally adjusted real aggregate expenditures on nondurable goods and services (source: U.S. Bureau of Economic Analysis, BEA). The deflators for aggregate nondurable and services consumption are different (source: BEA). Monthly dividends are measured by the real 12-month trailing dividends of the "NYSE/AMEX/NASDAQ" universe (source: Center for Research in Security Prices, CRSP) allowing for reinvestment at the gross risk free rates (source: CRSP). Inflation is calculated using the CPI (source: Federal Reserve Economic Data, FRED). Monthly consumption (dividend) growth is defined as log-differenced real consumption (dividend) per capita. The monthly population is obtained from the BEA. Monthly real market returns are the log value-weighted market return including dividends (source: CRSP, "NYSE/AMEX/NASDAQ"), minus inflation.[3] The sample spans the period between January

[2]In earlier versions, I use other macro-finance variables to approximate business conditions, including output growth, unemployment rate, and changes in the yield spread. In this version, I use the NBER recession indicator due to (1) simplicity and (2) consistency with the theoretical model that I show later.

[3]In earlier versions, I use Robert Shiller's monthly aggregate real 12-month trailing dividend data to conduct all the analyses in this research. Main results are robust. Tables, data, and codes are available upon request.

1959 and June 2014.

It is well-known that measured aggregate consumption data are flow data which are reported as total consumption over an extended period. This temporal aggregation results in a non-zero autoregressive coefficient of aggregate consumption growth (Working, 1960) even if the true consumption growth is i.i.d. The temporal aggregation effect could also potentially induce biases in the estimated conditional covariances, as thoroughly discussed in Duffee (2005, pp. 1691-1694). Therefore, I follow the literature and construct a measure of monthly consumption growth that removes the autoregressive terms up to the third order, $\Delta c_{t+1} - \sum_1^3 \phi_i(\Delta c_{t+1-i} - \overline{c})$ where ϕ_i is the i^{th}-order autoregressive coefficient and $\overline{c}$ is the sample mean. For the rest of this paper, "consumption growth" refers to this measure that controls for temporal aggregation.

The present research uses observed dividend data to identify stylized facts (this section) and provide exact data point estimates to be matched by a more general economic model (later). As a result, a one-time dividend event—unique and anticipated never to recur—should be excluded from the analysis because such an event is not drawn from the distribution relevant to current and future prices. In my sample period, I identify two extremely large dividend payments that significantly inflated dividend growth volatility and are considered unrepresentative: (1) The Microsoft special dividend payment in November 2004, and (2) the expiration of the Jobs and Growth Tax Relief Reconciliation Act of 2003 (known as the "The Bush Tax Cuts") on December 31, 2012 which incentivized a special dividend uptick during Q4 of 2012. I provide two observations that support the identification. First, while the 99th percentile of the real 12-month trailing dividend growth distribution is 3%, monthly dividend growth rates during November 2004 and December 2012 are 13% and 7%, respectively. Second, due to the 12-month trailing calculation, these two events result in the two lowest monthly dividend growth rates 12 months later, -7% in November 2005 and -4% in December 2013. Meanwhile, the 1st percentile of the distribution is -2.5%. To treat these two events, I linearly interpolate the corresponding CRSP-implied dividend data points using values before and after, prior to the 12-month trailing calculation.

2.3 Estimation and Results

The first step applies the maximum likelihood estimation (MLE) methodology to estimate the conditional variance models for each variable and selects the best model based on goodness of fit criteria. A detailed model selection analysis and parameter estimates are in the Internet Appendix. Using the standardized residuals obtained from the first step, the second step applies the quasi-maximum likelihood asymptotic theory to estimate the conditional correlation models. This section discusses the estimation results, with a focus on the cyclicalities of relevant second and cross moments pertinent to the Duffee Puzzle.

2.3.1 Conditional Variance Models

As shown in the Internet Appendix, the best univariate model of consumption growth variance is GED-GARCH with a countercyclical long-run mean q_t. The BEGE model with a strongly countercyclical long-run mean fits the market return variance the best, suggesting that market return innovations are more asymmetric than consumption growth innovations. The conditional variances of market return components exhibit different cyclical behaviors. On the one hand, the conditional variance of dividend growth is found to have a weakly procyclical long-run mean, as q_t appears to comove negatively with the NBER recession indicator. The best univariate model of dividend growth variance is GED-GARCH. On the other hand, there is strong evidence for countercyclical conditional variance of the valuation part of market returns.

2.3.2 Conditional Correlation Models and The Duffee Puzzle

I discuss two tables next. Table 1 reports the estimation results of the original DCC model and a cyclical DCC model (DCC-q_t). Table 2 reports the regression coefficients of conditional moments on a business cycle indicator.

Table 1 first shows evidence of a procyclical long-run mean in the consumption-return conditional correlation, given the significant and negative cyclicality coefficient estimate (ν=-0.1123, SE=0.0339). In terms of economic magnitudes, the consumption-return conditional correlation increases to around 0.1628 during non-recession periods and around 0.1124 during recession periods. The Likelihood Ratio (LR) test shows that the cyclical DCC model outperforms the original DCC model (p-value=0.0037).

Together with the conditional variance estimates of consumption growth and market returns, the consumption-return *covariance* estimates appear weakly procyclical, confirming the Duffee Puzzle. Using various business cycle indicators, the regression coefficient on consumption growth (consistent with asset pricing models) is 0.2963×10^{-3} (SE=0.1090×10^{-3}) and that on the NBER recession indicator is -0.0092×10^{-5} (SE=0.1019×10^{-5}). As expected, the procyclicality of the market return covariance is weaker than the procyclicality of the market return correlation because both market return and consumption growth volatilities are strongly countercyclical (see the first panel of Table 2). As robustness check, when GARCH estimates are used for both consumption and return conditional variances (as used in Duffee, 2005), the regression coefficient on consumption growth is 0.3201×10^{-3} (SE=0.1288×10^{-3}) and that on the NBER recession indicator is -0.1802×10^{-5} (SE=0.1201×10^{-5}).

Next, I provide empirical evidence for the source(s) of procyclicality in the Duffee Puzzle. When estimating the conditional correlation between dividend growth and consumption growth, the LR test rejects the DCC model in favor of the DCC-q_t model (p-value=0.0008; see the middle block of Table 1). The cyclicality coefficient estimate is significant and negative (ν=-0.1925, SE=0.1052), suggesting a procyclical consumption-dividend correlation. The average conditional correlation is 0.0049 during during recession periods and 0.0131 during non-recession periods.[4] Moreover, the consumption-dividend correlation, covariance, and the

[4]At the monthly frequency, the unconditional correlation of consumption and dividend growth is 0.0569 using the raw data, 0.0242 using de-centered residuals, and 0.0121 using de-centered standardized residuals. The quarterly unconditional correlation between consumption and dividend growth (calculated as the sum of monthly growth rates within the same quarter) is 0.1568. In an earlier version of the paper, I use Shiller's monthly dividend data. One can replicate his data in two steps: (1) obtain the 12-month trailing CRSP-implied dividend levels at the quarterly frequency, then (2) calculate dividends for other months using linear interpolation, e.g., April

sensitivity of dividend growth to consumption growth ("beta") all appear to be procyclical. As shown in Table 2, the regression coefficients on the NBER recession indicator "$b(I_{NBER,t})$" are -0.0082 (SE=0.0044), -0.0317$\times10^{-5}$ (SE=0.0173$\times10^{-5}$), and -0.0342 (SE=0.0209), respectively. In particular, a procyclical beta indicates that changes in aggregate dividend payments are more positively sensitive to contemporaneous consumption shocks during booms than during recessions. Table 2 also provides formal evidence for procyclical dividend growth volatility ($b(I_{NBER,t})$=-0.0009, SE=0.0003). All of the four procyclicality findings above—correlation, covariance, beta, and dividend volatility—are new to the literature.

As for the cyclical behavior of the valuation part of the puzzle, Table 2 suggests that the valuation covariance behaves countercyclically ($b(I_{NBER,t})$=0.2276$\times10^{-5}$, SE=0.1107$\times10^{-5}$). Instead of directly estimating the valuation covariance, one can also use the difference between the market return covariance estimates and the immediate cash flow covariance estimates. The countercyclicality result is robust to different estimates of valuation covariance, and both direct and indirect estimates are highly correlated (0.950 at the monthly frequency and 0.984 at the quarterly frequency). In addition, although the valuation conditional correlation is shown to be procyclical, countercyclical volatilities of consumption growth and the valuation part of market returns clearly dampen the procyclical valuation correlation.

In summary, I provide supportive evidence for the Duffee Puzzle—even after accounting for more sophisticated conditional variance models and adding 13 more years of data that now includes the 2007-08 financial crisis. Moreover, I have shown that the conditional covariance between dividend growth and consumption growth is a unique and consistent source of procyclicality in the puzzle, given the current return decomposition. This is the main empirical finding of the paper.

2.3.3 Time Series

Figure 1 depicts the decomposition of the market return covariance in the top plot, and the time-varying proportion of the immediate cash flow covariance in the bottom plot. The immediate cash flow covariance (solid blue line) has a statistically significant and negative correlation with the NBER recession indicator at -0.08 (SE=0.04). This covariance is smaller in magnitude and appears less persistent. The largest positive spike appeared in April 1984, coinciding with the largest drop in the monthly real consumption growth (-12.9 annualized percents[5]) and a large consumption-dividend correlation (0.25). The largest negative spike in this covariance appeared in December 1984 due to the continuing high consumption growth volatility and a small negative correlation. As robustness check, excluding year-months of 1984

= $\frac{2}{3}$March + $\frac{1}{3}$June and May = $\frac{1}{3}$March + $\frac{2}{3}$June, and so on. Due to this linear interpolation methodology, Shiller's monthly dividend growth series (essentially capturing information at the quarterly frequency) has a 0.18 correlation with the monthly consumption growth. Hence, his monthly dividend data is not suitable in the present research due to the overly smoothed monthly dividend growth.

[5] In early 1984, the nominal non-durable consumption level dropped significantly while its price index increased, which together resulted in the largest drop in real consumption growth during the sample period and a large increase in the consumption volatility. The Internet Appendix reports the time series plots of conditional volatilities of consumption growth and dividend growth.

and 1985 does not change the cyclicality result of the immediate cash flow covariance: The regression coefficient on the NBER recession indicator is -0.0302$\times 10^{-5}$ (SE=0.0155$\times 10^{-5}$), and their correlation remains significant and negative at -0.08.

On the other hand, the difference between the market return covariance and the immediate cash flow covariance (dashed red line) exhibits an overall countercyclical pattern, which is consistent with the evidence from Table 2. This covariance is more persistent with several salient spikes. For instance, the spike around 1987 in a generally decreasing trend corresponds to the Stock Market Black Monday in October 1987 that caused extreme price movements.[6]

One interesting implication of these findings is that the proportion of the immediate cash flow covariance in the Duffee Puzzle varies greatly over time. The market return covariance estimates drop below zero briefly during the 1960 recession (see Figure 2). This mechanically results in an extremely volatile cash flow proportion with abnormal values. As a result, I focus on the sample period from 1962 to 2014 in this paragraph. According to the bottom plot of Figure 1, the proportion is procyclical with an NBER regression coefficient of -0.0337 (SE=0.0124). This result provides supportive evidence for the core finding of the paper (i.e., consumption-dividend comovement is a consistent source of procyclicality in the Duffee Puzzle; see Section 2.3.2).

As for the magnitude of the proportion, the immediate cash flow covariance explains, on average, 5.00% of the total market covariance from 1962 to 2014. In particular, the proportion drops to 0% (even slightly negative) during recession periods, which is significantly lower than the 5.47% during non-recession periods (t stats=-2.72). It reached as high as 52.47% in the late 1980s, 50.60% in 2009 and 41.58% in the 2013 expansion, while it dipped below -50% consecutively during the first two months of 2007-08 recession. The unconditional proportion calculated using unconditional covariances is 7.89% using this subsample and 8.62 using the full sample.

2.4 Summary

Below, I summarize the cyclicalities of eight relevant second and cross moments established in this section:

(a).	The conditional variance of Δc is countercyclical.	*Kandel & Stambaugh (1990)*
(b).	The conditional variance of Δd is procyclical.	*New*
(c).	The conditional correlation between Δc and Δd is procyclical.	*New*
(d).	The conditional covariance between Δc and Δd is procyclical.	*New*
(e).	The conditional sensitivity (beta) of Δd to Δc is procyclical.	*New*
(f).	The conditional variance of $r^m - \Delta d$ is countercyclical.	*New*
(g).	The conditional variance of r^m is countercyclical.	*Schwert (1989)*
(h).	The conditional covariance between Δc and $r^m - \Delta d$ is countercyclical.	*New*

[6]Figure IA2 of the Internet Appendix presents the same plot but using the direct valuation covariance estimates, $Cov_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$. Both measures of valuation covariance are highly correlated as mentioned above (monthly: 0.950; quarterly: 0.984), and main time series properties remain the same.

3 A New DGP for the Joint Consumption-Dividend Dynamics

Extant consumption-based asset pricing theories do not typically discuss whether the modeling choice of aggregate dividends is realistic or not. In a Lucas tree economy (Lucas, 1978), dividends equal consumption, but the literature mostly models them as unit root processes with constant correlations.[7] Table 3 summarizes seven consumption-based asset pricing models and their abilities to match the empirical facts established in Section 2. Using suggested parameter choices, DGPs of these models fail to generate realistic dynamics of consumption-dividend comovement and dividend variance. Furthermore, these models imply either a countercyclical or zero market return covariance, which contradicts the Duffee Puzzle.

This section presents a new DGP that has the potential to accommodate Facts (a)–(e) into a consumption-based asset pricing model with a minimum number of state variables. To enhance the plausibility of the new DGP, I then discuss the economic interpretations of the state variables and shocks given the estimation results. In Appendix A, I discuss why alternative modeling approaches may be less suited to fit the salient empirical facts.

3.1 The DGP

The consumption growth, Δc_{t+1}, is assumed with a constant mean $\overline{c}$ and a composite shock structure:

$$\begin{aligned} \Delta c_{t+1} &= \overline{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, \\ \widetilde{\omega}_{c,t+1} &\sim N(0,1), \\ \widetilde{\omega}_{n,t+1} &\sim \Gamma(n_t, 1) - n_t. \end{aligned} \tag{9}$$

The "fundamental" consumption shock, $\widetilde{\omega}_{c,t+1}$, is a Gaussian shock with unit standard deviation; $\sigma_c > 0$ is a scale parameter. The "event" consumption shock, $\widetilde{\omega}_{n,t+1}$, follows a centered, positively-skewed, and heteroskedastic gamma distribution with a strictly positive shape parameter n_t. To model negative skewness in consumption growth, scale parameter σ_n is negative. Given the moment generating functions of Gaussian and gamma shocks, the conditional variance of Δc_{t+1}, denoted as $V_{c,t}$, is driven by n_t: $V_{c,t} = \sigma_c^2 + \sigma_n^2 n_t$. As a result, n_t is the macroeconomic uncertainty state variable in this DGP.

The dividend growth, Δd_{t+1}, has the following process:

$$\begin{aligned} \Delta d_{t+1} &= \overline{d} + \phi_d \left(V_{c,t} - \overline{V}_c \right) + b_t \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, \\ \widetilde{\omega}_{d,t+1} &\sim \Gamma(V_d, 1) - V_d. \end{aligned} \tag{10}$$

The expected dividend growth has a constant part ($\overline{d}$), and a time-varying part that decreases with macroeconomic uncertainty ($\phi_d < 0$); $\overline{V}_c$ denotes the mean of $V_{c,t}$. State variable b_t

[7] For example, Campbell and Cochrane (1999) assume constant comovement and variances. Bansal and Yaron (2004) assume a zero consumption-dividend conditional comovement given their shock assumption. Bansal, Kiku, and Yaron (2012) allow the dividend growth innovations to have a constant exposure to the consumption shock.

captures the time-varying sensitivity of dividend growth to consumption growth. The dividend-specific shock, $\widetilde{\omega}_{d,t+1}$, follows a centered, positively-skewed, and homoskedastic gamma distribution with a strictly positive shape parameter V_d. Scale parameter σ_d determines the sign of the dividend growth skewness.

The three shocks are mutually independent. Δc_{t+1} and Δd_{t+1} are observables. The two latent state variables, n_t and b_t, follow autoregressive processes that have positive exposures to the event and fundamental shocks, respectively (ϕ_n, ϕ_b, σ_{nn}, $\lambda_b > 0$):

$$n_{t+1} = (1-\phi_n)\overline{n} + \phi_n n_t + \sigma_{nn}\widetilde{\omega}_{n,t+1}, \quad (11)$$

$$b_{t+1} = (1-\phi_b)\overline{b} + \phi_b b_t + \lambda_b \sigma_c \widetilde{\omega}_{c,t+1}. \quad (12)$$

The key reason for this DGP to imply both countercyclical consumption growth variance and procyclical dividend growth variance and comovement is the flexible use of the two consumption shocks. The DGP exhibits time-varying volatility of consumption growth through *event* consumption shocks. Meanwhile, dividends react more to *fundamental* consumption shocks in booms (times when past consumption growth has been high). The cyclicalities of the two state variables can be proved:

■ Fact Check (a): *n_{t+1} is countercyclical, given $Cov_t(\Delta c_{t+1}, n_{t+1}) = \sigma_n \sigma_{nn} n_t < 0$.*
■ Fact Check (e): *b_{t+1} is procyclical, given $Cov_t(\Delta c_{t+1}, b_{t+1}) = \lambda_b \sigma_c^2 > 0$.*

It follows that the new DGP generates strictly procyclical consumption-dividend comovement and procyclical dividend growth variance:

■ Fact Check (b): *the conditional variance of dividend growth, $b_t^2\sigma_c^2 + \sigma_d^2 V_d$, is procyclical if $b_t > \frac{(\phi_b-1)\bar{b}}{\phi_b}$. (Note: $\frac{(\phi_b-1)\bar{b}}{\phi_b} < 0$)*
■ Fact Check (c): *the conditional correlation between dividend and consumption growth, $\frac{b_t\sigma_c^2}{\sqrt{\sigma_c^2+\sigma_d^2 n_t}\sqrt{b_t^2\sigma_c^2+\sigma_d^2 V_d}}$, is procyclical given a countercyclical n_t and a procyclical b_t.*
■ Fact Check (d): *the conditional covariance between dividend and consumption growth, $b_t\sigma_c^2$, is procyclical.*

3.2 DGP Estimation Results

Given that there is no feedback from the cash flow process to consumption, I estimate the consumption growth system $\{\Delta c, n\}$ and the dividend growth system $\{\Delta d, b\}$ in two separate steps using MLE-based methodologies. Appendix B details the estimation procedure.

The estimation results in Table 4 confirm that the new DGP matches Facts (a)–(e). Confirming Fact (a), Panel A shows that consumption growth depends negatively on the positively-skewed heteroskedastic event shock (σ_n=-0.0023, SE=0.0005), while the macroeconomic uncertainty state variable n_t has a positive exposure to the event shock (σ_{nn}=0.2772, SE=0.1027). Panel B then confirms Fact (e) that this new comovement state variable b_t is procyclical, as b_t loads significantly and positively on the fundamental shock (λ_b=59.9163, SE=5.4008). To provide direct evidence for these two facts, the regression coefficients of n_t and b_t on the NBER recession indicator are 0.5926 (SE=0.0354) and -0.1240 (SE=0.0190), respectively, as shown in Table 4. Similarly, confirming Facts (b)–(d), the DGP-implied dividend growth conditional variance, consumption-dividend conditional correlation, and consumption-dividend conditional covariance are procyclical given significant and negative NBER loadings.[8]

Figure 3 depicts the time series of the estimated DGP state variables: macroeconomic uncertainty n_t (top) and consumption-dividend comovement b_t (bottom). The monthly estimates of n_t exhibit a persistent process that occasionally spikes, mostly during recessions. Its countercyclical nature determines the countercyclicality of the consumption growth variance, $\sigma_c^2 + \sigma_n^2 n_t$. While σ_c^2 contributed by the fundamental shock explains on average 82% of the total consumption growth variance, $\sigma_n^2 n_t$ contributed by the event shock could explain as high as 34%–58% during recessions. On the other hand, as shown in the bottom plot of Figure 3, the consumption-dividend comovement state variable b_t is a less persistent process, which is consistent with observations from the empirical model in Section 2. The monthly b_t estimates have a significant and negative correlation of -0.25 with the NBER recession indicator.

3.3 Economic Interpretation of Consumption Shocks

The consumption shock structure plays an indispensable role in enabling the DGP to simultaneously satisfy Facts (a)–(e). In the following section, I motivate the economic interpretations of the two consumption shocks.

First, Table 5 documents that the filtered fundamental consumption shock $\tilde{\omega}_c$ is procyclical, given its significant and negative correlation with the NBER recession indicator at various frequencies (monthly: -0.18; quarterly: -0.27). The fundamental shock also comoves significantly and negatively with the detrended quarterly consumption-wealth ratio $\widehat{cay}$ from Lettau and Ludvigson (2001), which is consistent with the DGP. In this DGP, a unit fundamental shock at time t increases b_t and will have persistent and positive effects on the expected future consumption-dividend comovement and dividend variance. This variance becomes capitalized in financial wealth, inducing a higher asset price. The top plot of Figure 4 illustrates the negative relationship between the filtered fundamental shock $\tilde{\omega}_c$ (solid line) and $\widehat{cay}$ (dashed line) at the quarterly frequency. In addition, negative spikes in $\tilde{\omega}_c$ often appear during the NBER recessions. To increase the confidence about the shock assumption, Panel A of Table 5 shows that the filtered fundamental shock cannot be rejected by the standardized Gaussian shock assumption.

Second, as shown in the bottom plot of Figure 4, major positive spikes of the filtered event consumption shock $\tilde{\omega}_n$ occur during recessions. Because consumption growth loads negatively on the event shock (σ_n=-0.0023, SE=0.0005), these spikes reflect extreme negative consumption growth events. Table 5 formally confirms the countercyclicality of $\tilde{\omega}_n$, as evidenced by its correlation of 0.13 (0.25) with the monthly (quarterly) NBER recession indicator. In addition,

[8]Implied dividend growth volatility: $b(I_{NBER,t})$=-4.35$\times 10^{-6}$, SE=2.59$\times 10^{-6}$; implied covariance: $b(I_{NBER,t})$=-1.03$\times 10^{-6}$, SE=1.59$\times 10^{-7}$; implied correlation: $b(I_{NBER,t})$=-0.028, SE=0.004.

variable $\widehat{cay}$ appears uncorrelated with the filtered event shock $\tilde{\omega}_n$ and dividend-specific shock $\tilde{\omega}_d$. This result in turn supports the possibly close economic relationship between $\widehat{cay}$ and the fundamental shock, as discussed above.

It is noteworthy that several recent models in the consumption-based asset pricing literature have attempted to model consumption growth disturbance with two independent shocks. The continuous-time model in Longstaff and Piazzesi (2004) models the consumption growth innovation with a Brownian motion and a jump process which are conceptually similar to the Gaussian fundamental shock and the gamma event shock in my DGP. Closer to my model, the DGP in Bekaert and Engstrom (2017) features two independent heteroskedastic gamma shocks, one associated with the "good" volatility and the other with "bad" volatility. In a similar vein, Segal, Shaliastovich, and Yaron (2015) explore good and bad shocks in a long-run risk framework. However, none of these models accommodate realistic consumption-dividend comovement.

4 An External Habit Model

The theoretical model in this section explores how incorporating realistic joint dynamics of consumption growth and dividend growth into an endowment economy affects equilibrium stock price dynamics, thus potentially accommodating the Duffee Puzzle. Between the two puzzle components, the procyclical cash flow conditional covariance is immediately satisfied given the new DGP. However, different consumption-based asset pricing paradigms have different implications for the cyclicality of the valuation component of the puzzle. In particular, the external habit formation paradigm is suitable to structurally examine the Duffee Puzzle for the following reasons. First, Campbell and Cochrane (1999; CC) already naturally entails a countercyclical valuation covariance through risk aversion: The effect of consumption shocks on the equity valuation ratio is amplified when risk aversion is higher. Second, as I show later, an endowment economy with procyclical dividend risk requires a countercyclical price of risk to generate procyclical equity prices. Third, this paradigm implies an equity premium that equals the product of time-varying price of risk and amount of risk, which is consistent with Duffee's (2005) original theoretical motivation for studying market return covariance.

Section 4.1 introduces the model. Section 4.2 derives (approximate) analytical model solution and implications. Section 4.3 confronts the numerical model solution with a wide range of empirical moments, featuring the eight stylized facts related to the Duffee Puzzle.

4.1 Pricing Kernel, Risk Free Rate, Sensitivity Function

I obtain the log real pricing kernel using the external habit preference as in the CC model:

$$m_{t+1} = \ln \beta - \gamma \Delta c_{t+1} - \gamma \Delta s_{t+1}. \quad (13)$$

The log surplus consumption dynamics incorporates the new consumption growth innovation,

$$s_{t+1} = (1-\phi_s)\overline{s}_t + \phi_s s_t + \lambda_t \left(\sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}\right), \tag{14}$$

where ϕ_s is the persistence coefficient, $\overline{s}_t$ the time-varying long-run mean, and λ_t the sensitivity function.

The real risk free rate, rf_t, is solved from the first-order condition for the consumption-saving choice, $rf_t = \ln E_t[\exp(m_{t+1})]^{-1}$. Given the moment generating functions of the two independent shocks in the pricing kernel ($\widetilde{\omega}_{c,t+1}$, $\widetilde{\omega}_{n,t+1}$), the risk free rate has an exact closed-form solution,

$$rf_t = -\ln\beta + \gamma\overline{c} \underbrace{+\gamma(1-\phi_s)(\overline{s}_t - s_t)}_{\text{intertemporal substitution}} \underbrace{-\frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \left[\gamma(1+\lambda_t)\sigma_n - \ln\left(1+\gamma(1+\lambda_t)\sigma_n\right)\right] n_t}_{\text{precautionary savings}}. \tag{15}$$

As in the CC model, the intertemporal substitution effect and the precautionary savings effect counteract in determining the time variation in the risk free rate. The literature has proposed various modeling choices of the sensitivity function.[9] The present model proposes a strictly procyclical real rate, which is consistent with the few empirical findings such as Ang, Bekaert, and Wei (2008). Specifically, λ_t is chosen such that the second-order Taylor approximation of the risk free rate is a constant as in the CC model:

$$\lambda_t = \begin{cases} \frac{1}{\overline{S}_t}\sqrt{1-2(s_t-\overline{s}_t)} - 1, & s_t \leq s_{max,t} \\ 0, & s_t > s_{max,t} \end{cases} \tag{16}$$

where $\overline{s}_t = \log(\overline{S}_t)$ and $s_{max,t}$ are derived as functions of the free parameters and n_t,

$$\overline{S}_t = \sqrt{(\sigma_c^2 + \sigma_n^2 n_t)\frac{\gamma}{1-\phi_s}}, \tag{17}$$

$$s_{max,t} = \overline{s}_t + \frac{1}{2}(1-\overline{S}_t^2). \tag{18}$$

$\overline{S}_t$ and $s_{max,t}$ are time-varying equivalent to those in the CC model. The dynamics of the sensitivity function are determined by s_t and n_t. As in the CC model, when the consumption level is closer to the habit level, or lower s_t, the sensitivity function increases. On the other hand, the uncertainty state variable n_t has a negative effect on λ_t through $\frac{1}{\overline{S}_t}$ and a positive effect through $\overline{s}_t$.

With this sensitivity function, the precautionary savings channel in the risk free rate contains a higher-order moment, which is different from the CC model. For instance, a third-

[9]For instance, in the CC model, the two effects offset each other, resulting in a constant risk free rate. Wachter (2005, 2006) allow the intertemporal substitution effect to dominate in order to generate an upward sloping real yield curve, thus resulting in a countercyclical short rate. Bekaert and Engstrom (2017) proposed a time-varying risk free rate such that the relative importance of the two effects varies over time, depending on the magnitudes of their good and bad uncertainty state variables.

order Taylor approximation of the risk free rate is given by:

$$rf_t \approx \underbrace{-\ln\beta + \gamma\bar{c} - \frac{(1-\phi_s)\gamma}{2}}_{\equiv rf_{CC}} + \frac{1}{3}\gamma^3(1+\lambda_t)^3 \underbrace{\sigma_n^3}_{<0} n_t. \tag{19}$$

rf_{CC} denotes the constant risk free rate as in the CC model (which assumes only Gaussian shocks). The appended precautionary savings term, $\frac{1}{3}\gamma^3(1+\lambda_t)^3\sigma_n^3 n_t$, is strictly procyclical given $\sigma_n < 0$. It captures that, under extreme and bad economic environment, the desire to save might eventually dominate the intertemporal substitution effect, resulting in a lower rate. Appendix C provides the derivations and graphical demonstrations of the sensitivity function.

4.2 Approximate Analytical Solution

This model features three state variables: the procyclical log surplus consumption ratio (s_t), the countercyclical macroeconomic uncertainty (n_t), and the procyclical consumption-dividend comovement (b_t).[10] The model does not have an exact closed-form solution. In this section, I explore economic intuitions based on an approximate analytical solution.

4.2.1 Equity Prices

I conjecture an approximate process for the log valuation ratio $pd_t \equiv \ln\left(\frac{P_t}{D_t}\right)$:

$$pd_t = A_0 + A_1 s_t + A_2 b_t + A_3 b_t^2 + A_4 n_t. \tag{20}$$

Then, I apply the Campbell–Shiller linearization to the log market return, $r_{t+1}^m = \ln\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) \approx \Delta d_{t+1} + a_1 pd_{t+1} - pd_t + a_0$ where a_0 and a_1 are linearization constants. Given the shock assumptions and the pd conjecture, there are three types of shocks in this approximate log market return: Gaussian shocks, $\chi^2(1)$ shocks, and gamma shocks. I then apply a quasi quadratic Taylor approximation to the Euler equation. $E_t\left[\exp(m_{t+1} + r_{t+1}^m)\right]$ can be shown to have the following expression, $\exp\left[E_t(m_{t+1} + r_{t+1}^m) + \frac{1}{2}V_t(m_{t+1} + r_{t+1}^m)\right]$; see Appendix D for the proof. The coefficients in the conjectured log valuation ratio are solved in closed form by equating the terms of the state variables; see Appendix E for the technical details and proofs.

I focus on the asset pricing implications of the procyclical comovement state variable introduced in this paper, b_t. Through a pure cash flow (CF) effectl, the valuation ratio can be interpreted as reflecting the outlook on future dividend growth, given that the expected value of the exponential of dividend growth increases with both the expected growth and conditional variance.[11] In this model, the persistent procyclical consumption-dividend comovement induces

[10] The cyclicality of each state variable can be easily proved. Log surplus consumption ratio is procyclical because $Cov_t(s_{t+1}, \Delta c_{t+1}) = \lambda_t(\sigma_c^2 + \sigma_n^2 n_t) > 0$. As discussed in Section 3, macroeconomic uncertainty is countercyclical because $Cov_t(n_{t+1}, \Delta c_{t+1}) = \sigma_n \sigma_{nn} n_t < 0$, and consumption-dividend comovement is procyclical because $Cov_t(b_{t+1}, \Delta c_{t+1}) = \lambda_b \sigma_c^2 > 0$.

[11] The Gaussian analogue is, $E_t\left[\exp(\Delta d_{t+1})\right] = \exp\left[E_t(\Delta d_{t+1}) + \frac{1}{2}V_t(\Delta d_{t+1})\right]$, where $E_t(\Delta d_{t+1})$ is $\bar{d} + \phi_d(V_{c,t} - \overline{V})$ and $\frac{1}{2}V_t(\Delta d_{t+1})$ is $\frac{1}{2}(b_t^2\sigma_c^2 + \sigma_d^2)$.

a persistent procyclical dividend growth variance. This variance becomes capitalized in stock prices. Therefore, this pure CF effect suggests a positive relationship between b_t^2 and pd_t.

In addition, there is a risk premium effect. The total risk premium to compensate changes in dividend growth can be approximated with $-Cov_t(m_{t+1}, \Delta d_{t+1}) = \gamma(1+\lambda_t)b_t\sigma_c^2$. This compensation increases with both the price-of-risk variable λ_t and the consumption-dividend comovement variable b_t. When a positive fundamental shock arrives, b_t and s_t increase and λ_t decreases simultaneously. If λ_t was not countercyclical, the model would generate a higher risk premium and a lower asset price, yielding a counterintuitive negative relationship between the procyclical b_t and the procyclical pd_t. However, the strongly countercyclical price of risk in this habit formation model is able to dominate and generate a positive pd_t–b_t relationship.

PROPOSITION 1. *Given appropriate parameter choices, there exist positive comovement effects on the equity valuation ratio, $A_2, A_3 > 0$.*

As for the other two state variables, the surplus consumption ratio effect as in CC implies a positive A_1. Different from CC, there are competing effects that determine A_4. The CF effect of uncertainty is well-understood: When macroeconomic uncertainty (n_t) increases, future dividend growth is expected to drop, driving down the current price. However, a higher n_t also induces more precautionary savings, driving down the interest rate and lowering the total return demanded. This discount rate (DR) effect of uncertainty also appears in Bekaert and Engstrom (2017). Given appropriate parameter choices, the CF effect dominates the DR effect when extreme event shocks occur, yielding a negative A_4; in other times, A_4 becomes positive.

■ Fact Check (f) and (g): *Given the dividend growth dynamics in the new DGP and the valuation ratio conjecture, the conditional variances of the log valuation ratio and the log market return have the following approximate expressions:*

$$\begin{aligned} Var_t(pd_{t+1}) &\approx \varsigma_{pd} + \varsigma_1\lambda_t + \varsigma_2 b_t + \varsigma_3 n_t + \varsigma_4\lambda_t^2 + \varsigma_5 b_t^2 + \varsigma_6\lambda_t b_t + \varsigma_7\lambda_t n_t + \varsigma_8\lambda_t^2 n_t, \\ Var_t(r_{t+1}^m) &\approx \varsigma_{rm} + a_1^2\varsigma_1\lambda_t + \left[a_1^2\varsigma_2 + 2a_1\lambda_b\sigma_c^2\left(A_2 + 2A_3(1-\phi_b)\bar{b}\right)\right]b_t + a_1^2\varsigma_3 n_t + a_1^2\varsigma_4\lambda_t^2 \\ &+ \left(a_1^2\varsigma_5 + 2a_1\varsigma_2 + \sigma_c^2\right)b_t^2 + \left(a_1^2\varsigma_6 + 2a_1\varsigma_1\right)\lambda_t b_t + a_1^2\varsigma_7\lambda_t n_t + a_1^2\varsigma_8\lambda_t^2 n_t, \end{aligned}$$

where ς_{pd}, ς_{rm}, ς_1, ς_2, ς_3, ς_4, ς_5, ς_6, ς_8, a_1, λ_b, σ_c, ϕ_b and $\bar{b}$ are strictly positive constants, and $\varsigma_7 = 2A_1A_4\sigma_n\sigma_{nn}$ is positive when the cash flow effect of n_t dominates the discount rate effect, and negative vice versa. The model has the potential to generate countercyclical variances if the procyclical terms are counteracted by the countercyclical terms.

4.2.2 The Total Amount of Risk and the Duffee Puzzle

The approximate analytical solution of the total amount of risk is as follows:

$$\begin{aligned} & \underbrace{b_t\sigma_c^2}_{\text{[1]. Immediate cash flow covariance: dividend risk}} \\ + & \underbrace{a_1A_1\lambda_t\sigma_c^2}_{\text{[2]. Valuation covariance: benchmark time-varying amount of risk as in CC}} \\ + & \underbrace{\left[a_1A_2\lambda_b + 2a_1A_3(1-\phi_b)\bar{b}\lambda_b + 2a_1A_3\phi_b\lambda_b b_t\right]\sigma_c^2}_{\text{[3]. Valuation covariance: comovement risk}} \\ + & \underbrace{a_1\left[A_1\lambda_t\sigma_n^2 + A_4\sigma_{nn}\sigma_n\right]n_t}_{\text{[4]. Valuation covariance: downside risk and uncertainty risk}}, \end{aligned} \tag{21}$$

where parameters $\sigma_c, \sigma_{nn}, \bar{b}, \phi_b, \lambda_b$ and a_1 are positive and σ_n is negative according to the DGP estimation results in Table 4.

Term [1] captures the procyclical immediate cash flow covariance $Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$, or the amount of dividend risk. Meanwhile, the other three terms constitute the valuation covariance component of the Duffee Puzzle. Specifically, Term [2] captures the amount of risk implied from linearizing the original CC model (with a Gaussian consumption shock and constant volatility σ_c) and is strictly countercyclical. Term [3] captures the procyclical amount of dividend comovement risk through the valuation channel, which is new to the literature. Term [4] captures the amount of risk that is associated with the countercyclical macroeconomic uncertainty, and implies that the cyclicality of Term [4] is state-dependent. At a higher risk aversion state, the coefficient of macroeconomic uncertainty "$\left[A_1\lambda_t\sigma_n^2 + A_4\sigma_{nn}\sigma_n\right]$" is more likely to be positive, rendering Term [4] countercyclical.

In summary, the new procyclical consumption-dividend comovement state variable induces two new procyclical terms in the amount of risk: dividend risk (via cash flows) and comovement risk (via valuation). This analytical solution thus suggests the ability of this model to potentially accommodate the Duffee Puzzle in a habit formation framework.

■ Fact Check (h): *The model has the potential to generate a countercyclical valuation covariance if the procyclical terms are counteracted by the countercyclical terms.*

4.2.3 The Equity Premium

The equity premium in this approximate analytical solution is expressed as the product of a countercyclical price of risk $\gamma(1+\lambda_t)$—which is consistent with CC—and a time-varying amount of risk that now comprises both procyclical and countercyclical terms according to Equation (21). These terms exhibit countervailing effects on the magnitude of the equity premium. On the one hand, the introduction of the countercyclical uncertainty state variable n_t makes the asset riskier, since both the time-varying uncertainty and price of risk are higher during economic turmoil. From this perspective, a higher unconditional equity premium is expected. On the other hand, the introduction of the procyclical comovement state variable b_t lowers the level of the unconditional equity premium. This is because the amount of risk now contains procyclical terms which counteract the countercyclical amount-of-risk terms and the countercyclical price of risk. As a result, the asset becomes less risky.

Moreover, in contrast to the CC model, the conditional equity premium no longer monotonically increases when consumption drops. This is because there are two types of consumption shocks in the economy. For instance, a negative *event* shock increases both the price of risk and the amount of risk, whereas a negative *fundamental* shock increases the price of risk while lowering the amounts of dividend risk and comovement risk. This fundamental channel cannot be ignored because, empirically, this fundamental shock accounts for more than 80% of the total consumption variance in a long sample (see discussions in Section 3.2). Therefore, the ultimate impact of a consumption shock on the conditional equity premium can be nonlinear. Appendix E provides the derivations.

4.3 Numerical Solution

To identify the implications of each of the three state variables, I conduct an overlaying numerical analysis. The baseline model, referred to as M(1) in the rest of the paper, is an adapted Campbell and Cochrane (1999) model that features homoskedastic fundamentals, constant consumption-dividend comovement, and time-varying surplus consumption ratio (the only state variable). Then, M(2), building on M(1), is an adapted Bekaert and Engstrom (2017) model that incorporates countercyclical macroeconomic uncertainty as the second state variable. Finally, my model, labeled as M(3), overlays M(2) with procyclical consumption-dividend comovement in the dividend process. Appendix C provides mathematical descriptions of M(1) and M(2). All three models price dividend claims.

Section 4.3.1 describes the calibration of the non-DGP parameters. Then, I evaluate the fit of the Duffee Puzzle moments in Section 4.3.2 and the fit of conventional unconditional asset moments in Section 4.3.3.

4.3.1 Calibration and Simulation

Table 6 summarizes the four non-DGP parameters. The utility curvature parameter γ is fixed at 2. As commonly assumed in the literature, the AR(1) coefficient of the s_t process, ϕ_s, equals the AR(1) coefficient of monthly log valuation ratio. The benchmark constant risk free rate, rf_{CC}, as appeared in Equation (19), is chosen to match the average monthly real short rate (proxied by the difference between the change in log nominal 90-day Treasury index constructed by CRSP and the continuously compounded inflation rate). β is the time discount parameter inferred from the rf_{CC} equation.

The log valuation ratios are solved numerically using the "series method" from Wachter (2005). M(1) is solved using a one-dimensional grid (20×1) for the one state variable: the log surplus consumption ratio. M(2) is solved over a two-dimensional grid (20×20) for the two state variables: the log surplus consumption ratio and macroeconomic uncertainty. The final model M(3) uses a three-dimensional grid (20×20×20) for all three state variables. Then, for each model, I simulate the shocks for 100,000 months given their distributional assumptions and parameter estimates, and construct the state variable processes accordingly. Given the grid solutions, I apply the piecewise polynomial cubic interpolation for M(1), and the piecewise polynomial spline interpolation for M(2) and M(3) to obtain the log valuation ratio for each simulated month given the state variable values. All the reported theoretical moments in this paper are calculated using the second half of the simulated dataset.

4.3.2 Fitting the Duffee Puzzle

Table 7 examines the closeness between the empirical and simulation asset moments of Facts (a)–(h), using recession and non-recession subsamples. One challenge is to identify *realistic* recessions in a simulated consumption-based economy, as this research focuses on variables' cyclical behaviors. For this purpose, I develop an algorithm based on the simulated consumption growth such that the algorithm mechanically mimics the identification of NBER recessions that are based on patterns in GDP growth; details and empirical tests are provided in Appendix F.

On fitting Facts (a)–(e), most simulation moment point estimates of both recession and non-recession periods in M(3) are within 95% confidence intervals of the actual data point estimates. The only exception is the recession-period dividend growth volatility, $\sigma(\Delta d)$ ($I_{rece.} = 1$), due to the excessively volatile conditional mean of the dividend growth in simulating n_t.[12] Nevertheless, row "$\sigma_t(\Delta d_{t+1}) \sim I_{rece.,t}$" in Table 7 confirms that the *conditional* dividend growth variance is strictly procyclical. M(3) by design outperforms M(1) and M(2). While most simulation moments of M(1) and M(2) are not rejected by the empirical counterparts, they fail to fit all of the cyclical moments.

On fitting Facts (f) and (g), M(1)—an adapted CC model—generates recession and non-recession volatilities of $r^m - \Delta d$ and r^m that are significantly lower than the empirical counterparts. Meanwhile, M(2) and M(3), which allow for countercyclical consumption growth uncertainty n_t, improve the fit with realistic magnitudes. It is noteworthy that the new comovement state variable b_t in M(3) dampens the volatilities of $r^m - \Delta d$ and r^m. As suggested by the analytical solution in Equation (21), the procyclical dividend risk and comovement risk counteract the countercyclical terms in the amount of risk. Assets in M(3) become less risky: When consumption drops during recessions, asset prices do not drop as much as those in M(2).

On fitting Fact (h), M(1) and M(2) tend to generate non-recession valuation covariance that is too high, e.g., 4.0956×10^{-5} in M(1) and 3.1658×10^{-5} in M(2) versus 1.7436×10^{-5} in data. The reason why M(1) generates a high valuation covariance despite the low price variation is that there is a high correlation (around 0.7 in my simulation) between the price dynamics and consumption growth innovations through the surplus consumption ratio. Such correlation decreases to around 0.25 in M(2) due to adding a second state variable to the price dynamics. Despite the misfit of M(2) in terms of matching the magnitudes, allowing

[12] Gamma shocks (e.g. $\tilde{\omega}_n$) are more likely to obtain extreme values than Gaussian shocks in a simulation; in this context, it could result in a more volatile latent process of n_t and thus a more volatile conditional mean of dividend growth that is a linear function of n_t.

for countercyclical macroeconomic uncertainty clearly generates more countercyclicality in the valuation covariance. The difference between recession and non-recession valuation covariance is wider in M(2) than in M(1). In M(3), one major improvement to M(2) is that both recession and non-recession valuation covariances are now lower and statistically closer to the data point estimates. This is largely because of the lower price variability as shown in row (f). Therefore, M(3) fits Fact (h) from both the perspectives of cyclicality and magnitude.

This overlaying numerical analysis so far demonstrates that price dynamics become different after introducing the new procyclical comovement state variable in M(3). Finally, as shown in the last two rows in Table 7, M(3) is the only model (among the three) that simulates point estimates of recession and non-recession amount of consumption risk that *cannot* be rejected by data. In contrast, M(1) and M(2) generate amount of consumption risk point estimates that are higher than data point estimates. As a result, M(3) potentially addresses the Duffee Puzzle by generating a more realistic magnitude of consumption risk.[13]

As overidentification tests, Table 8 evaluates the fit of the three models in terms of 17 unconditional moments including the eight Duffee-Puzzle moments: $\sigma(\Delta c)$, $\sigma(\Delta d)$, $\rho(\Delta d, \Delta c)$, $C(\Delta d, \Delta c)$, $b(\Delta d, \Delta c)$, $\sigma(r^m - \Delta d)$, $\sigma(r^m)$, $C(r^m - \Delta d, \Delta c)$. As shown in the first 13 rows, despite the different DGP assumptions among the three overlaying models, all their unconditional simulation moments are shown to match the data generally well, which hence is not an informative evaluation. M(1) generates significantly smaller unconditional volatilities of $r^m - \Delta d$ and r^m but significantly larger unconditional $C(r^m - \Delta d, \Delta c)$ and $C(r^m, \Delta c)$ than data. On the other hand, M(2) and M(3) are not rejected by data in terms of these 4 moments. The magnitude of the unconditional return-consumption covariance in M(3), 3.1738×10^{-5}, is the closest to its data counterpart, 2.4822×10^{-5}.

4.3.3 Conventional Moments

Finally, Table 9 reports the fit of the models with respect to a set of conventional unconditional moments. Incorporating the procyclical comovement state variable, M(3) implies a slightly lower unconditional equity premium, 5.6524%, than M(2), 6.2414%. This is consistent with the economic intuition mentioned in Section 4.2.3: The amount of risk in M(3) now contains procyclical immediate cash flow and comovement risks, resulting in a less risky asset. In addition, consistent with the literature, the introduction of countercyclical macroeconomic uncertainty state variable clearly increases the equity premium significantly, from 3.8751% in M(1) to 6.2414% in M(2). The uncertainty state variable overall introduces additional countercyclical dynamics into the amount of risk, resulting in a more risky asset and a lower average valuation ratio (which is also shown in Table 9).

Because the procyclical comovement state variable contributes positively to the valuation ratio, M(3) implies volatility of the valuation ratio at 0.2594, which is the highest and the closest

[13]Note that the non-recession point estimate of return-consumption covariance (1.8546×10^{-5}) is smaller in magnitude than the recession estimate (3.3894×10^{-5}) in data, although both are statistically indifferent according to the t test. The slightly higher recession-sample estimate is expected because the conditional means of both return and consumption growth series are expected to comove more during recessions.

to the data point estimate among the three models. The market return volatility implied by M(3) is slightly smaller, 14.7234%, than that implied by M(2), which can be explained analytically as follows. Unconditional market return variance can be roughly decomposed into three components: unconditional variances and covariance of changes in pd and Δd. In particular, according to the law of iterated expectations, $Cov(a_1 pd_{t+1} - pd_t, \Delta d_{t+1}) = E\left[Cov_t(pd_{t+1}, \Delta d_{t+1})\right]$ contains a $E(A_1 \lambda_t \bar{b} \sigma_c^2)$ term in M(1) and M(2) but a $E(A_1 \lambda_t b_t \sigma_c^2)$ term in M(3) which contains an additional negative covariance term, $Cov(\lambda_t, b_t) < 0$. This reflects the non-linear effect of a fundamental consumption shock in asset prices, through procyclical comovement risk and through countercyclical risk aversion simultaneously.

The implied Sharpe Ratio from M(3), 0.3888, is the closest to the data moment because of the smaller implied equity premium. Moreover, the kurtosis moment is matched statistically well by all three models. M(2) and M(3) generate the same risk free rate dynamics as they only differ in the cash flow part; this average risk free rate is statistically close to the data counterpart.

5 Conclusion

This paper aims to understand and accommodate the Duffee Puzzle in consumption-based asset pricing models. To achieve these aims, I first show empirically that the conditional covariance between the immediate cash flow part of market returns (dividend growth) and consumption growth is (1) procyclical and (2) a consistent source of procyclicality in the puzzle. This is the core empirical finding of the paper. Then, I device a new DGP that is able to simultaneously accommodate procyclical consumption-dividend comovement and countercyclical consumption growth volatility. Finally, I solve a variant of the Campbell and Cochrane model incorporating this new DGP. The approximate analytical solution suggests that the procyclical consumption-dividend comovement, as a new state variable, induces two new procyclical terms in the amount of consumption risk: dividend risk (via cash flows) and comovement risk (via valuation). These procyclical terms, according to the numerical solution, play a crucial role in generating realistic magnitude of the amount of risk. Furthermore, the model, after accommodating the Duffee Puzzle, implies a lower equity premium because stock prices now incorporate procyclical risks, rendering the asset less risky.

This paper is agnostic about why dividends and consumption comove procyclically. One potential explanation is the phenomenon documented in Lintner (1956) and Brav, Graham, Harvey, and Michaely (2005) that managers are more likely to smooth dividend payments during bad times, causing changes in financial payouts to be less associated with macroeconomic shocks. Regardless of the source, the cash flow side of the aggregate economy merits further research.

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Appendices

A On the uniqueness of the DGP in Section 3

In this appendix section, I discuss why alternative modeling approaches for dividend and consumption growth shocks may be less suited to fit the salient empirical facts (a)–(e). The key challenge is to imply both procyclical cash flow-related moments (variance and comovement) and countercyclical consumption growth variance. My DGP achieves it by assuming two consumption shocks: the fundamental shock enters the dividend growth process with a procyclical exposure, while the event shock determines the heteroskedasticity of consumption growth. Alternatively, one could assume a "constant" exposure of dividend growth to the consumption fundamental shock—what is typically assumed in the literature—but a "procyclical" fundamental shock conditional variance. This way, during each period, the consumption growth disturbance is driven by a heteroskedastic Gaussian shock with procyclical volatility and a heteroskedastic gamma shock with countercyclical volatility. With proper parameter values, this model can generate procyclical consumption-dividend comovement and countercyclical consumption variance. However, this alternative DGP has two potential problems. First, the identification of consumption growth variance is likely to be quite difficult. The analytical expression of the consumption growth conditional variance is now the sum of procyclical and countercyclical components. Given that a Gaussian distribution is symmetric and not bounded, a heteroskedastic Gaussian fundamental shock might act as the event shock trying to fit the left-tail events in the estimation. This likely results in countercyclical volatility of fundamental shock. Granted, one can restrict the fundamental shock volatility to be procyclical by restricting signs of certain parameters; however, it is difficult to interpret results of a constrained estimation. Second, besides the estimation difficulties, other empirical facts will be immediately violated. For instance, the implied consumption-dividend comovement (due to the constant exposure) becomes countercyclical. In addition, while the data finds significant and negative correlation (-0.2090) between consumption and dividend growth variances, this alternative DGP will generate a strictly positive correlation between consumption and dividend growth variances because of the constant and positive exposure.

B Estimation procedure for the new DGP in Section 3

Given that there is no feedback from the dividend growth process to the consumption growth process, I conduct a two-step estimation procedure. The first step estimates the consumption growth system. I use a filtration-based maximum likelihood methodology in Bates (2006) to estimate the latent macroeconomic uncertainty state variable n_t and the two consumption shocks, the fundamental shock $\widehat{\widetilde{\omega}}_{c,t+1}$ and the event shock $\widehat{\widetilde{\omega}}_{n,t+1}$ where "$\widehat{\ }$" indicates the estimated variables. The conditional consumption growth variance and its long-run average are then obtained, $\widehat{V}_{c,t}$ and $\widehat{\overline{V}}_{c,t}$.

The second step takes the dividend growth data Δd_{t+1} and state variable and shock estimates from the first step $\{\widehat{V}_{c,t}, \widehat{\overline{V}}_{c,t}, \widehat{\sigma}_c, \widehat{\widetilde{\omega}}_{c,t+1}\}$. To provide estimation convenience, dividend growth is first projected onto $\widehat{V}_{c,t} - \widehat{\overline{V}}_{c,t}$ to obtain the estimates for $\{\overline{d}, \phi_d\}$. The rest of the dividend growth system is then estimated by maximizing the sum of the log likelihoods of the implied cash flow-specific shock $\widetilde{\omega}_{d,t+1}$. The MLE estimation does not impose constraints on the non-negativity of b_t estimates, but imposes one constraint to ensure a valid gamma density function for $\widetilde{\omega}_{d,t+1}$ at any time stamp t. If $\widetilde{\omega}_{d,t+1}$ in the data is more right-tailed and is bounded below, then σ_d will be estimated to be positive and the constraint is as follows: $-\sigma_d V_d \le \min_{\forall t \in 1,\dots,T}\left(\Delta d_{t+1} - \widehat{\overline{d}} - \widehat{\phi}_d(\widehat{V}_{c,t} - \widehat{\overline{V}}_{c,t}) - b_t \widehat{\widetilde{\omega}}_{c,t+1}\right)$. On the other hand, if $\widetilde{\omega}_{d,t+1}$ is more left-tailed and is bounded above, then σ_d will be estimated to be negative with the following constraint: $-\sigma_d V_d \ge \max_{\forall t \in 1,\dots,T}\left(\Delta d_{t+1} - \widehat{\overline{d}} - \widehat{\phi}_d(\widehat{V}_{c,t} - \widehat{\overline{V}}_{c,t}) - b_t \widehat{\widetilde{\omega}}_{c,t+1}\right)$.

C Intermediate models in Section 4

In this appendix section, I provide details of the two intermediate models in the overlaying framework. M(1) is an adapted CC model with constant macroeconomic uncertainty and consumption-dividend comovement, while M(2) builds on M(1) and allows for time-varying macroeconomic uncertainty. The DGPs of the models are as follows:

$$
\begin{aligned}
\text{M(1):}\ & \Delta c_{t+1} = \overline{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, && \text{(C.1)}\\
& \widetilde{\omega}_{c,t+1} \sim N(0,1), \widetilde{\omega}_{n,t+1} \sim \Gamma(\overline{n},1) - \overline{n}, && \text{(C.2)}\\
& \Delta d_{t+1} = \overline{d} + \overline{b}\sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, && \text{(C.3)}\\
& \widetilde{\omega}_{d,t+1} \sim \Gamma(V_d,1) - V_d, && \text{(C.4)}\\
& \overline{c} = 0.0025, \sigma_c = 0.0029, \sigma_n = -0.0023, \overline{n} = 0.3742, \\
& \overline{d} = 0.0015, \sigma_d = 0.000123, V_d = 8933.5172, \overline{b} = 0.0944;\\
\text{M(2):}\ & \Delta c_{t+1} = \overline{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, && \text{(C.5)}\\
& n_{t+1} = (1-\phi_n)\overline{n} + \phi_n n_t + \sigma_{nn} \widetilde{\omega}_{n,t+1}, && \text{(C.6)}\\
& \widetilde{\omega}_{c,t+1} \sim N(0,1), \widetilde{\omega}_{n,t+1} \sim \Gamma(n_t,1) - n_t, && \text{(C.7)}\\
& \Delta d_{t+1} = \overline{d} + \phi_d(V_{c,t} - \overline{V}_c) + \overline{b}\sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, && \text{(C.8)}\\
& V_{c,t} = Var_t\left(\Delta c_{t+1}\right) = \sigma_c^2 + \sigma_n^2 n_t, \overline{V}_c = E\left(V_{c,t}\right), && \text{(C.9)}\\
& \widetilde{\omega}_{d,t+1} \sim \Gamma(V_d,1) - V_d, && \text{(C.10)}\\
& \overline{c} = 0.0025, \sigma_c = 0.0029, \sigma_n = -0.0023, \overline{n} = 0.3742, \phi_n = 0.9500, \sigma_{nn} = 0.2772, \\
& \overline{d} = 0.0015, \phi_d = -630.8768, \sigma_d = 0.000123, V_d = 8933.5172, \overline{b} = 0.0944;
\end{aligned}
$$

M(3): The new DGP in this paper (as shown in Table 4).

The log surplus consumption ratios in all three models follow an AR(1) process with time-varying sensitivities to the consumption growth innovation. The sensitivity functions are chosen as follows:

$$
\lambda_t = \begin{cases} \frac{1}{\overline{S}_t}\sqrt{1-2(s_t-\overline{s}_t)} - 1, & s_t \leq s_{max,t} \\ 0, & s_t > s_{max,t} \end{cases}, \tag{C.11}
$$

$$
\overline{s}_t = \log(\overline{S}_t), \tag{C.12}
$$

$$
s_{max,t} = \overline{s}_t + \frac{1}{2}(1-\overline{S}_t^2), \tag{C.13}
$$

$$
\text{M(1):}\ \overline{S}_t = \sqrt{(\sigma_c^2 + \sigma_n^2 \overline{n})\frac{\gamma}{1-\phi_s}}, \tag{C.14}
$$

$$
\text{M(2/3):}\ \overline{S}_t = \sqrt{(\sigma_c^2 + \sigma_n^2 n_t)\frac{\gamma}{1-\phi_s}}. \tag{C.15}
$$

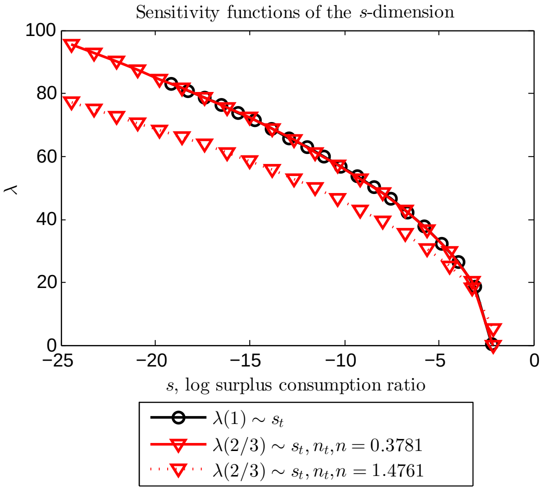
Given the sensitivity functions, the real risk free rates are time-varying with a higher moment appended to reflect the non-Gaussian nature of the consumption event shock:

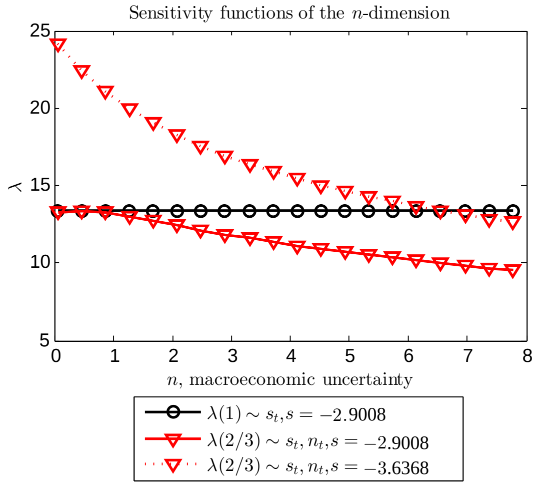
$$
\begin{aligned}
\text{M(1):}\ rf_t &= -\ln\beta + \gamma\overline{c} + \gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \left[\gamma(1+\lambda_t)\sigma_n - \ln\left(1+\gamma(1+\lambda_t)\sigma_n\right)\right]\overline{n} \\
&\approx -\ln\beta + \gamma\overline{c} + \underbrace{\gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_n^2\overline{n}}_{\text{fix}=-\frac{(1-\phi_s)\gamma}{2}} + \frac{1}{3}\gamma^3(1+\lambda_t)^3\sigma_n^3\overline{n}; \qquad \text{(C.16)}
\end{aligned}
$$

$$
\begin{aligned}
\text{M(2/3):}\ rf_t &= -\ln\beta + \gamma\overline{c} + \gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \left[\gamma(1+\lambda_t)\sigma_n - \ln\left(1+\gamma(1+\lambda_t)\sigma_n\right)\right]n_t \\
&\approx -\ln\beta + \gamma\overline{c} + \underbrace{\gamma(1-\phi_s)(\overline{s}_t - s_t) - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_c^2 - \frac{1}{2}\gamma^2(1+\lambda_t)^2\sigma_n^2 n_t}_{\text{fix}=-\frac{(1-\phi_s)\gamma}{2}} + \frac{1}{3}\gamma^3(1+\lambda_t)^3\sigma_n^3 n_t. \qquad \text{(C.17)}
\end{aligned}
$$

(C.18)

The calibration plots of the sensitivity functions are as follow:





D Quadratic approximation of the moment generating function of a random variable that is a linear combination of Gaussian, χ^2, and gamma shocks

Suppose a random variable x receives three independent shocks,

$$\begin{aligned} &x = \mu + x_1\omega + x_2(\omega^2 - 1) + x_3(\varepsilon - \alpha), \\ &\omega \sim N(0,1), \\ &\omega^2 \sim \chi^2(1), \\ &\varepsilon \sim \Gamma(\alpha, 1), \end{aligned} \tag{D.1}$$

where μ is the unconditional mean of variable x, and $\{x_t, x_2, x_3\}$ are constant coefficients. Recall the moment generating function (MGF) is $mgf_\omega(\nu) = \exp(\nu^2/2)$ for a standard Gaussian shock, $mgf_{\omega^2}(\nu) = (1-2\nu)^{-1/2}$ for a χ^2 shock, and $mgf_\varepsilon(\nu) = (1-\nu)^{-\alpha}$ for a gamma shock with a unit scale parameter and shape parameter equal to α. Therefore, the MGF of x, $mgf_x(\nu) = E[\exp(\nu x)]$, is as follows,

$$\begin{aligned} mgf_x(\nu) &= \exp(\nu\mu)E_t[\exp(\nu x_1\omega + \nu x_2(\omega^2 - 1) + \nu x_3(\varepsilon - \alpha))] \\ &= \exp(\nu\mu - \nu x_2 - \nu x_3\alpha) mgf_\omega(\nu x_1) mgf_{\omega^2}(\nu x_2) mgf_\varepsilon(\nu x_3) \\ &= \exp(\nu\mu - \nu x_2 - \nu x_3\alpha)\exp\left\{\frac{1}{2}(\nu x_1)^2\right\}(1-2\nu x_2)^{-1/2}(1-\nu x_3)^{-\alpha} \\ &= \exp(\nu\mu - \nu x_2 - \nu x_3\alpha)\exp\left\{\frac{1}{2}(\nu x_1)^2 - \frac{1}{2}\ln(1-2\nu x_2) - \alpha\ln(1-\nu x_3)\right\}. \end{aligned} \tag{D.2}$$

It can be easily shown that the quadratic approximation to $\ln(1-z)$ is $-z - \frac{1}{2}z^2$. The quadratic approximations to $mgf_x(\nu)$ yields:

$$\begin{aligned} mgf_x(\nu) &\approx \exp(\nu\mu - \nu x_2 - \nu x_3\alpha)\exp\left\{\frac{1}{2}(\nu x_1)^2 + \nu x_2 + (\nu x_2)^2 + \nu x_3\alpha + \frac{1}{2}(\nu x_3)^2\alpha\right\} \\ &= \exp(\nu\mu)\exp\left\{\frac{1}{2}(\nu x_1)^2 + (\nu x_2)^2 + \frac{1}{2}(\nu x_3)^2\alpha\right\} \\ &= \exp(\nu E(x))\exp\left\{\frac{1}{2}\nu^2 V(x)\right\}. \end{aligned} \tag{D.3}$$

Define $X = \exp(x)$ and set $\nu = 1$,

$$E(X) \approx \exp\left\{E(x) + \frac{1}{2}V(x)\right\}. \tag{D.4}$$

E Approximate analytical solution of M(3)

In this appendix section, I solve the theoretical model in Section 4 with an approximate analytical solution. There are three approximations. The first approximation conjectures the log valuation ratio pd_t, $pd_t = A_0 + A_1 s_t + A_2 b_t + A_3 b_t^2 + A_4 n_t$. The second approximation applies the Campbell–Shiller linearization to the log market return, $r^m_{t+1} = \Delta d_{t+1} + a_1 pd_{t+1} - pd_t + a_0$. The log market return can be approximately expressed as a linear function of the state variables and four independent shocks to the economy:

$$
\begin{aligned}
r^m_{t+1} = &\overline{d} - \phi_d \sigma_d^2 \overline{n} + a_1\left(A_0 + A_1(1-\phi_s)\overline{s} + A_2(1-\phi_b)\overline{b} + A_3((1-\phi_b)\overline{b})^2 + A_4(1-\phi_n)\overline{n}\right) - A_0 + a_0 \\
&+ A_1(a_1\phi_s - 1)s_t + \left(a_1 A_2 \phi_b + 2a_1 A_3 (1-\phi_b)\overline{b}\phi_b - A_2\right) b_t \\
&+ A_3(a_1\phi_b^2 - 1)b_t^2 + \left(a_1 A_4 \phi_n - A_4 + \phi_d \sigma_d^2\right) n_t \\
&+ \left(a_1 A_1 \lambda_t + a_1 A_2 \lambda_b + 2a_1 A_3(1-\phi_b)\overline{b}\lambda_b + (1 + 2a_1 A_3 \phi_b \lambda_b) b_t\right) \sigma_c \widetilde{\omega}_{c,t+1} \\
&+ a_1 A_3 \sigma_c^2 \lambda_b^2 \left(\widetilde{\omega}_{c,t+1}\right)^2 + a_1\left(A_1 \lambda_t \sigma_n + A_4 \sigma_{nn}\right) \widetilde{\omega}_{n,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}.
\end{aligned} \tag{E.1}
$$

The third approximation applies quadratic approximation to the MGF of random variable $m_{t+1} + r^m_{t+1}$ using the proof in Appendix D. With the approximate logarithm of the Euler equation and by equating the terms for the state variables, the coefficients in the valuation ratio equation are solved:

$$A_1 = \frac{\gamma(1-\phi_s)}{1 - a_1\phi_s} > 0, \tag{E.2}$$

$$A_2 = \frac{(1 + 2a_1 A_3 \phi_b \lambda_b)\left[\gamma(1+\lambda_t) - (a_1 A_1 \lambda_t + 2a_1 A_3 (1-\phi_b)\overline{b}\lambda_b)\right]\sigma_c^2 - 2a_1 A_3 (1-\phi_b)\phi_b \overline{b}}{a_1\phi_b - 1 + a_1\lambda_b(1 + 2a_1 A_3 \phi_b \lambda_b)\sigma_c^2} > 0. \tag{E.3}$$

$$A_3 = \frac{-2a_1\phi_b\lambda_b\sigma_c^2 + 1 - a_1\phi_b^2 \pm \sqrt{(2a_1\phi_b\lambda_b\sigma_c^2 - 1 + a_1\phi_b^2)^2 - 4a_1^2\phi_b^2\lambda_b^2\sigma_c^4}}{4a_1^2\phi_b^2\lambda_b^2\sigma_c^2} > 0. \tag{E.4}$$

$$A_4 = \frac{\xi_t \pm \sqrt{\xi_t^2 - 2\sigma_{nn}^2 a_1^2\left(\phi_d\sigma_d^2 + \frac{1}{2}\left(A_1\lambda_t a_1 - \gamma(1+\lambda_t)\right)^2\sigma_n^2\right)}}{2\sigma_{nn}^2 a_1^2} > 0, \tag{E.5}$$

$$\xi_t = 1 - \phi_n a_1 - a_1^2 A_1 \lambda_t \sigma_n \sigma_{nn} + \gamma(1+\lambda_t) a_1 \sigma_n \sigma_{nn}. \tag{E.6}$$

An approximate analytical solution of equity premium is derived, given the quadratic approximation:

$$
\begin{aligned}
E_t\left(r^m_{t+1}\right) - rf_t + \frac{1}{2}Var_t\left(r^m_{t+1}\right) &\approx -Cov_t\left(r^m_{t+1}, m_{t+1}\right) \\
&= \underbrace{\gamma(1+\lambda_t)}_{\text{price of consumption risk}} \\
&\times \Big\{\underbrace{a_1 A_1 \lambda_t \sigma_c^2}_{\text{(1). approximate amount of consumption risk in CC}} \\
&+ \underbrace{\left[a_1 A_2 \lambda_b + 2a_1 A_3(1-\phi_b)\overline{b}\lambda_b + (1 + 2a_1 A_3\phi_b\lambda_b)b_t\right]\sigma_c^2}_{\text{(2). additional amount of consumption risk induced by comovement}} \\
&+ \underbrace{a_1\left[A_1\lambda_t\sigma_n^2 + A_4\sigma_{nn}\sigma_n\right]n_t\Big\}}_{\text{(3). additional amount of consumption risk induced by uncertainty}} .
\end{aligned} \tag{E.7}
$$

F Identifying recessions using simulated monthly consumption growth

I develop an algorithm to identify realistic recessions based on the simulated consumption growth rates such that the algorithm mechanically mimics the NBER recession indicator which is based on the quarterly GDP growth rates. Here is a proposed algorithm that identifies recessions ex post:

1. *Quarterly Growth*: Aggregate the monthly consumption growth into a quarterly frequency;

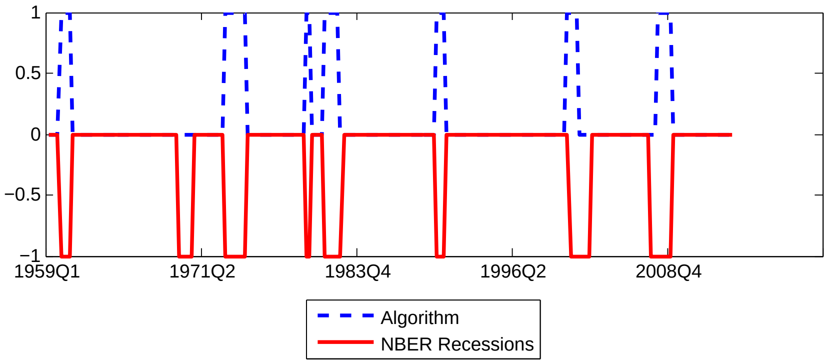
2. *Standardization*: De-center the quarterly consumption growth by a 49-quarter moving average using 24 quarters before and after (24+1+24), and divide it with its long-term or unconditional standard deviation to obtain the standardized consumption growth rates;

3. *Fundamental Cyclical Events*: Identify the recession quarters if there are at least two consecutive standardized consumption growth drops that are <-0.9;

4. *Extreme Cyclical Events*: For an extreme event (standardized consumption growth $<= -2$), if its *immediate adjacent* quarters before and/or after exhibit negative standardized growths, then the extreme event and its adjacent quarter(s) are considered as recession quarters. Given the immediate adjacent quarter after (before) the extreme event, if the next (previous) quarters with consecutive standardized consumption growth <-0.9, they are also considered recession quarters. The purpose is to capture extreme events which have a buildup before and a persistent effect after.

5. *Trough Points but with Positive Growths*: Given the recessions identified in Steps 3 and 4, if there is a recession period lasting for at least three quarters and the following quarter has a positive growth rate, then this quarter is also considered a recession period. The reason is that positive growth rates could be obtained mechanically because of the low denominator from the previous period.

To increase the plausibility of this algorithm, I apply this algorithm to actual consumption growth data from January 1959 to June 2014. This algorithm is able to identify seven out of the eight NBER recessions; regressing the consumption-based recession indicator on the NBER recession indicator yields a coefficient of 0.9038 (SE=0.0507), which is statistically close to 1; interestingly, without Step 5, the regression coefficient is 0.8795 (SE=0.0558). The plot below compares the consumption-based recessions (dashed blue) and the NBER recession indicator (solid red). Data and codes for applications are available upon requests.



G Dependences of the valuation ratio on s, n, and b

The three plots depict the dependences of PD on the three state variables, respectively, for all three models. M(1): valuation ratio depends on s only and is depicted in solid black lines with circles. M(2): valuation ratio is sensitive to s and n and is depicted in solid red lines with diamonds; M(3): valuation ratio is sensitive to s, n and b and is depicted in blue lines with squares. The dimension is reduced by fixing the other state variables at their mean values (mean of s: -2.9008; mean of n: 0.3781; mean of b: 0.0942) and critical values (e.g., 5% percentile in s: -3.6368; 95% percentile in n: 1.4761; 95% percentile in b: 0.3932). Hence, the lines in the plots can be interpreted with a conditional statement.

In the top plot, the positive association between PD and the comovement state variable b confirms the analytical prediction. The M(1) and M(2) horizontal lines intersect the $E(s)$–$E(n)$ plane of M(3) at around $b = 0.094$, which is expected because $\bar{b} = 0.094$ according to Table 4. The convex increasing pattern indicates that the impact of b on asset prices is stronger when the level of consumption-dividend comovement is higher. This is consistent with my replication of the model in an earlier version of the paper, where I calibrate a procyclical b with an average around 0.35; in that economy, the amount of consumption risk is significantly (more) procyclical and the immediate cash flow part even yields 20% of the total variation—More information is available upon request. In the present calibration, the mean (and in fact median) of the simulated b_t is around 0.1 and its 95th value is 0.39.

The valuation ratio implied by M(3) at the "lower 5th s"–"$E(n)$" plane ($s = -3.6368, n = 0.3781$) lays below the "$E(s)$"–"$E(n)$" plane ($s = -2.9008, n = 0.3781$), indicating a positive relationship between PD and s; this is confirmed by the left bottom plot. Similarly, the valuation ratio at the "$E(s)$"–"higher 95th n" plane ($s = -2.9008, n = 1.4761$) is above the $E(s)$–$E(n)$ plane, indicating a positive relationship between PD and n; this is confirmed by the right bottom plot. It is noteworthy that the average and 95th percentile of the simulated n_t, 0.3781 and 1.4761 respectively, are within the lower region in the bottom plot where the DR effect still dominates the traditional CF effect (see discussions in Section 4.2.1); thus, a positive relationship between PD and n is expected. However, the hump shape is interesting as it captures that, under extremely high macro uncertainty, the CF effect dominates and stock prices decrease with uncertainty.

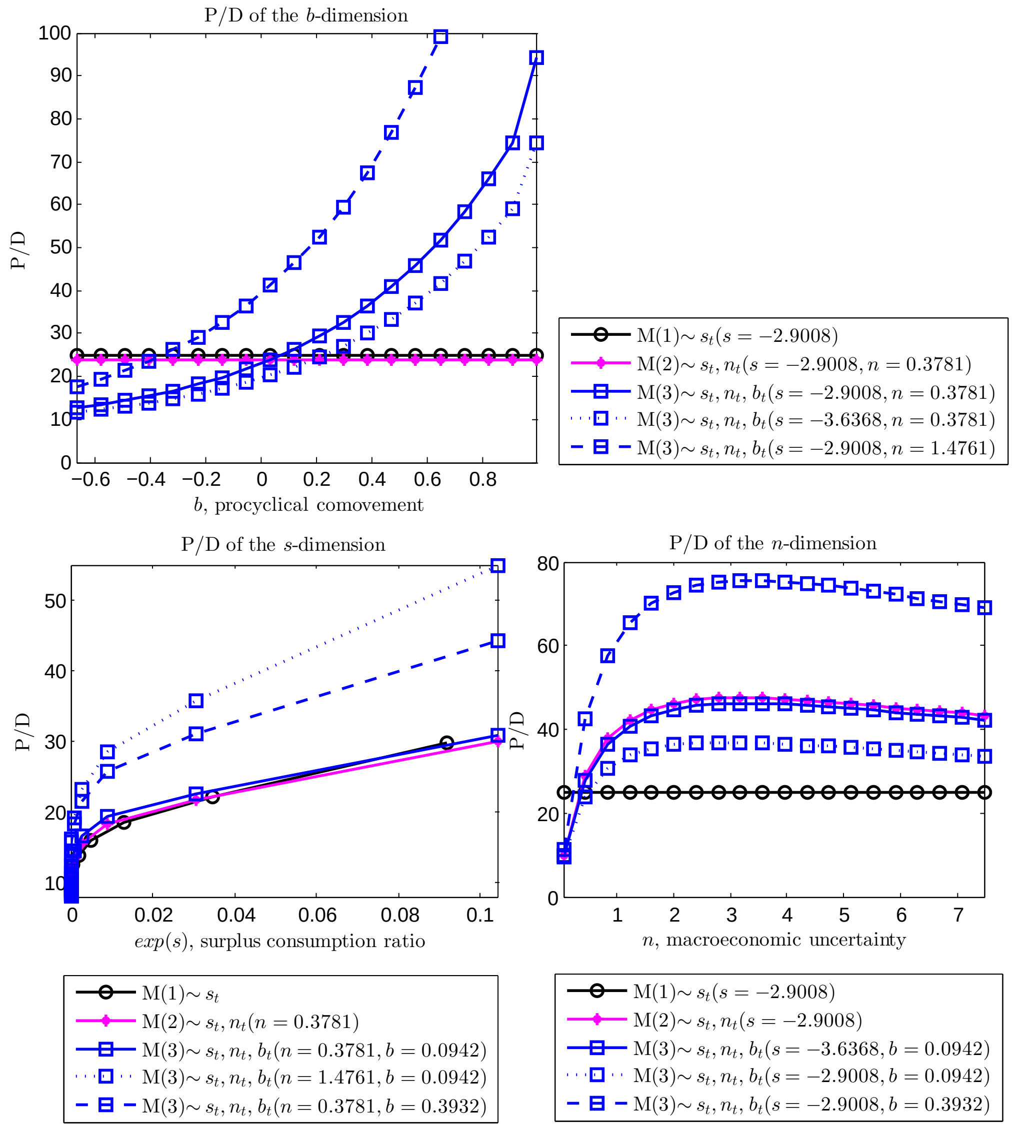


Table 1: Conditional Correlations between Consumption Growth and Market Return Components.

This table presents the estimation results of DCC and a cyclical DCC model (DCC-q_t) using the standardized innovations of consumption growth and market return components. Model: The conditional correlation matrix $\boldsymbol{Corr_t}$ is modeled with a quadratic form, $(\boldsymbol{Q_t^*})^{-1}\boldsymbol{Q_t}(\boldsymbol{Q_t^*})^{-1}$, where $\boldsymbol{Q_t^*}$ is the diagonal matrix with the square root of the diagonal element of $\boldsymbol{Q_t}$ on the diagonal. The off-diagonal element of $\boldsymbol{Corr_t}$ is the conditional correlation of the standardized residuals, $\boldsymbol{z_{t+1}} \equiv [z_{1,t+1}, z_{2,t+1}]'$. The DCC-$q_t$ model is as follows,

$$\boldsymbol{Q_t} = \overline{Q}_{12}\begin{bmatrix} 1 & 1+q_t \\ 1+q_t & 1 \end{bmatrix} + \alpha_{12}\left[\boldsymbol{z_t z_t'} - \overline{Q}_{12}\begin{bmatrix} 1 & 1+q_{t-1} \\ 1+q_{t-1} & 1 \end{bmatrix}\right] + \beta_{12}\left[\boldsymbol{Q_{t-1}} - \overline{Q}_{12}\begin{bmatrix} 1 & 1+q_{t-1} \\ 1+q_{t-1} & 1 \end{bmatrix}\right],$$

where parameter $\overline{Q}_{12}$ is the off-diagonal term of the predetermined constant conditional correlation matrix $\frac{1}{T}\sum_{t=1}^{T}\boldsymbol{z_t z_t'}$; q_t is modeled as $\nu_{12}SNBER_t$, where $SNBER_t$ is a standardized NBER recession indicator. The DCC model is the null hypothesis of the DCC-q_t model. Other Notations: "LL", loglikelihood; the last two rows report the likelihood ratio test statistics and its p-value. Robust standard errors are shown in parentheses. Values in bold (italics) are statistically significant at a significant level of 5% (10%). N=665 months (1959/02∼2014/06).

Series 1:	*Consumption Growth (Δc)*					
Series 2:	*Market Return (r^m)*		*Dividend Growth (Δd)*		$r^m - \Delta d$	
	DCC	DCC-q_t	DCC	DCC-q_t	DCC	DCC-q_t
$\overline{Q}_{12}$	0.1558	0.1558	0.0121	0.0121	0.1508	0.1508
	(fix)	(fix)	(fix)	(fix)	(fix)	(fix)
α_{12}	0.0162	0.0171	0.0449	0.0437	0.0155	0.0161
	(0.0115)	(0.0112)	(0.0458)	(0.0472)	(0.0111)	(0.0107)
β_{12}	**0.9476**	**0.9457**	*0.3219*	**0.3320**	**0.9516**	**0.9512**
	(0.0178)	(0.0190)	(0.1956)	(0.1594)	(0.0166)	(0.0175)
ν_{12}		**-0.1123**		*-0.1925*		**-0.1096**
		(0.0339)		(0.1052)		(0.0333)
LL	662.49	666.70	671.48	677.08	663.09	667.74
N(param)	2	3	2	3	2	3
LR test stats. (H0=DCC)	-	8.41	-	11.19	-	9.30
P-value	-	0.0037	-	0.0008	-	0.0023

Table 2: Cyclical Moments in the Empirical Model: Decompose the Duffee Puzzle.

This table provides evidence on the cyclicalities of all second and cross moments pertinent to the Duffee Puzzle by regressing these conditional moments on the NBER recession indicator. Notations: Conditional volatility, $\sigma_t(x_{t+1})$; conditional covariance, $Cov_t(x_{t+1}, y_{t+1})$; conditional correlation, $\frac{Cov_t(x_{t+1},y_{t+1})}{\sigma_t(x_{t+1})\sigma_t(y_{t+1})}$; and conditional beta, $\frac{Cov_t(x_{t+1},y_{t+1})}{\sigma_t^2(x_{t+1})}$; "$b(I_{NBER})$", regression coefficient; "SE", standard error; "Counter-", countercyclical; "Pro-", procyclical. Given the magnitude of covariances, regression coefficients and SEs are scaled up by 10^5 for reporting purposes. Values in bold are statistically significant at a one-sided significant level of 5%. . N=665 months (1959/02∼2014/06).

1. Volatility:	$\sigma_t(\Delta c_{t+1})$	$\sigma_t(r^m_{t+1})$	$\sigma_t(\Delta d_{t+1})$	$\sigma_t(r^m_{t+1} - \Delta d_{t+1})$
$b(I_{NBER,t})$	**1.50E-04**	**0.0164**	**-0.0009**	**0.0205**
SE	(6.86E-05)	(0.0004)	(0.0003)	(0.0009)
t	2.18	36.74	-3.46	21.99
	Counter-cyclical	Counter-	Pro-	Counter-
2. Duffee Puzzle:	$Cov_t(r^m_{t+1}, \Delta c_{t+1})$	$Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$	$Cov_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$	
$b(I_{NBER,t})$ ($\times 10^5$)	-0.0092	**-0.0317**	**0.2276**	
SE ($\times 10^5$)	(0.1019)	(0.0173)	(0.1107)	
t	-0.09	-1.84	2.06	
	Pro-	Pro-	Counter-	
3. Duffee Puzzle, Extensions:	$Corr_t(r^m_{t+1}, \Delta c_{t+1})$	$Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$	$Corr_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$	
$b(I_{NBER,t})$	**-0.0541**	**-0.0082**	**-0.0460**	
SE	(0.0063)	(0.0044)	(0.0062)	
t	-8.56	-1.89	-7.45	
	Pro-	Pro-	Pro-	
	$\beta_t(r^m_{t+1}, \Delta c_{t+1})$	$\beta_t(\Delta d_{t+1}, \Delta c_{t+1})$	$\beta_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$	
$b(I_{NBER,t})$	-0.1630	**-0.0342**	0.1517	
SE	(0.1160)	(0.0209)	(0.1314)	
t	-1.40	-1.64	1.15	
	Pro-	Pro-	Counter-	

Table 3: Seven Extant Consumption-Based Asset Pricing Models.

This table summarizes the cyclicality of Duffee Puzzle moments across seven well-cited variants of habit-formation and long-run risk models. (1) "CC1999": Campbell and Cochrane (1999, JPE); (2) "BEX2009": Bekaert, Engstrom, and Xing (2009, JFE) allow time variation in both uncertainty and risk aversion in square root-type processes; (3) "BE2017": Bekaert and Engstrom (2017, JPE) decompose aggregate consumption innovation into asymmetric gamma shocks but still assume constant exposures of dividend growth; (4) "BY2004": Bansal and Yaron (2004, JF); (5) "BTZ2009": Bollerslev, Tauchen, and Zhou (2009, RFS) assume a time-varying volatility of volatility in a square root-type process that is shown in closed form as the state variable driving the time variation in equity variance premium; (6) "BKY2012": Bansal, Kiku, and Yaron (2012, CFR) are the first to model a positive comovement between consumption growth and dividend growth under the LRR setting; (7) "SSY2015": Segal, Shaliastovich, and Yaron (2015, JFE) introduce asymmetric consumption shocks in a long-run risk framework. Column "Data" refers to the stylized facts presented in Table 2. "Const." indicates a constant moment, "Counter-" countercyclical, and "Pro-" procyclical (based on correlations with consumption growth).

	Data	(1) CC1999 Habit	(2) BEX2009 Habit	(3) BE2017 Habit	(4) BY2004 LRR	(5) BTZ2009 LRR	(6) BKY2012 LRR	(7) SSY2015 LRR
(a). $Var_t(\Delta c_{t+1})$	Counter-	Const.	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)
(b). $Var_t(\Delta d_{t+1})$	Pro-	Const.	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Const.
(c). $Cov_t(\Delta d_{t+1}, \Delta c_{t+1})$	Pro-	Const.	Counter-	Counter-	0	Counter- (⊛)	Counter- (⊛)	0
(d). $Corr_t(\Delta d_{t+1}, \Delta c_{t+1})$	Pro-	0.2	Const.	Unclear	0	1	Const.	0
(e). $\beta_t(\Delta d_{t+1}, \Delta c_{t+1})$	Pro-	Const.	Const.	Pro- (∗)	0	Const.	Const.	0
(f). $Var_t(r^m_{t+1} - \Delta d_{t+1})$	Counter-	Counter-	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)
(g). $Var_t(r^m_{t+1})$	Counter-	Counter-	Counter-	Counter-	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)
(h). $Cov_t(r^m_{t+1} - \Delta d_{t+1}, \Delta c_{t+1})$	Counter-	Counter-	Counter-	Counter-	0	0	0	Counter- (⊛)
(Duffee). $Cov_t\,(r^m_{t+1}, \Delta c_{t+1})$	Pro-	Counter-	Counter-	Counter-	0	Counter- (⊛)	Counter- (⊛)	Counter- (⊛)

(∗) Procyclical when the scale parameter of bad uncertainty shock in the total consumption shock (σ_{cn}) is greater than the scale parameter of bad uncertainty in dividend (σ_{dn}) in BE2017.
(⊛) Countercyclical when the time-varying consumption volatility is modeled to be countercyclical; note that the time-varying volatility is a crucial feature of LRR models; however, the LRR models do not imply countercyclical volatility because the volatility shock and the consumption shock are assumed uncorrelated.

Table 4: The New DGP for the Joint Consumption-Dividend Dynamics.

Consumption and dividend growth have the following joint dynamics:

$$\begin{aligned}
\Delta c_{t+1} &= \bar{c} + \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_n \widetilde{\omega}_{n,t+1}, \\
n_{t+1} &= (1-\phi_n)\overline{n} + \phi_n n_t + \sigma_{nn} \widetilde{\omega}_{n,t+1}, \\
\Delta d_{t+1} &= \overline{d} + \phi_d \left(V_{c,t} - \overline{V}_c\right) + b_t \sigma_c \widetilde{\omega}_{c,t+1} + \sigma_d \widetilde{\omega}_{d,t+1}, \\
b_{t+1} &= (1-\phi_b)\overline{b} + \phi_b b_t + \lambda_b \sigma_c \widetilde{\omega}_{c,t+1}, \\
V_{c,t} &= \sigma_c^2 + \sigma_n^2 n_t, \\
\overline{V}_c &= \sigma_c^2 + \sigma_n^2 \overline{n},
\end{aligned}$$

where the consumption fundamental shock $\widetilde{\omega}_{c,t+1} \sim N(0,1)$, the consumption event shock $\widetilde{\omega}_{n,t+1} \sim \Gamma(n_t, 1) - n_t$, and the dividend-specific shock $\widetilde{\omega}_{d,t+1} \sim \Gamma(V_d, 1) - V_d$. The DGP estimation adopts a two-step procedure and uses the AR(3)-de-meaned consumption growth and the original dividend growth as Δc_{t+1} and Δd_{t+1} (see details in Appendix B). Panels A and B present the estimation results; "ADF Test" reports the augmented Dickey-Fuller test statistics with the null that latent state variables n_{t+1} and b_{t+1} follow unit root processes; "$b(I_{NBER,t})$" reports the sensitivity of the state variables to the NBER recession indicator, which leads to the cyclicality result. Standard errors are shown in parentheses. Values in bold are statistically significant at a significant level of 5%. N=665 months (1959/02∼2014/06).

Panel A. Estimation Results, Consumption				Panel B. Estimation Results, Dividend			
	Δc_{t+1}		n_{t+1}		Δd_{t+1}		b_{t+1}
$\bar{c}$	**0.0025**	$\bar{n}$	**0.3742**	$\bar{d}$	**0.0015**	$\bar{b}$	0.0944
	(0.0001)		(0.1609)		(0.0004)		(0.1612)
σ_c	**0.0029**	ϕ_n	**0.9500**	ϕ_d	**-630.8768**	ϕ_b	**0.3159**
	(0.0001)		(0.0264)		(225.7119)		(0.1561)
σ_n	**-0.0023**	σ_{nn}	**0.2772**	σ_d	**1.23E-04**	λ_b	**59.9163**
	(0.0005)		(0.1027)		(3.36E-06)		(5.4008)
				V_d	**8933.5172**		
					(488.4230)		
		ADF Test	**-4.298**			ADF Test	**-18.775**
		$b(I_{NBER,t})$	**0.5926**			$b(I_{NBER,t})$	**-0.1240**
		Cyclicality	Counter-			Cyclicality	Pro-

Table 5: Properties of DGP Shocks.

Panel A presents the summary statistics of the three filtered monthly shocks from the DGP. Panels B reports the correlation between the monthly and quarterly shocks and business cycle indicators; quarterly shocks use the average of monthly shocks within the quarter; business cycle indicators include the NBER recession indicator and the detrended $\widehat{cay}$ measure from Lettau and Ludvigson (2001). Bootstrapped standard errors are reported in parentheses. Values in bold are statistically significant at a significant level of 5%. N=665 months (1959/02∼2014/06).

Panel A. Summary statistics				Panel B. Correlation w/ BC			
	$\tilde{\omega}_c$	$\tilde{\omega}_n$	$\tilde{\omega}_d$		$\tilde{\omega}_c$	$\tilde{\omega}_n$	$\tilde{\omega}_d$
Mean	1.71E-03	2.42E-03	1.66E-04	I_{NBER}, monthly	**-0.18**	**0.13**	**-0.11**
	(0.04)	(0.02)	(3.62)		(0.04)	(0.04)	(0.04)
Standard Deviation	**0.97**	**0.44**	**94.47**				
	(0.03)	(0.06)	(3.87)		$\tilde{\omega}_c^Q$	$\tilde{\omega}_n^Q$	$\tilde{\omega}_d^Q$
Scaled Skewness	0.18	**5.44**	0.19	I_{NBER}, quarterly	**-0.27**	**0.25**	**-0.23**
	(0.13)	(0.57)	(0.25)		(0.07)	(0.07)	(0.07)
Excess Kurtosis	0.45	**40.52**	**2.70**	$\widehat{cay}$, quarterly	**-0.22**	0.06	0.01
	(0.50)	(8.38)	(0.56)		(0.06)	(0.06)	(0.07)

Table 6: Non-DGP Model Parameter Choices (∗=annualized).

This table presents the non-DGP parameter choices and the derived parameter values. The AR(1) coefficient of s_t (ϕ_s) is obtained from the AR(1) coefficient of the monthly log valuation ratio; rf_{CC} is the constant benchmark risk free rate and is chosen to match the average real 90-day Treasury bill rate, which is proxied by changes in log nominal 90-day Treasury index (source: CRSP) minus inflation rate (source: FRED) continuously compounded; β is the time discount parameter derived from the rf_{CC} equation. Monthly data covers the period 1959/01-2014/06.

		Notation	Value
1. Non-DGP parameters:			
	Curvature parameter	γ	2
∗	s_t persistence	ϕ_s	0.9236
∗	Risk free rate (%)	rf_{CC}	1.4854
2. Derived parameters:			
∗	Discount rate	β	0.9694
	Steady-state surplus consumption ratio, M(1)	$\bar{S}$	0.0559
	Maximum log surplus consumption ratio, M(1)	s_{max}	-2.3863

Table 7: Theoretical Models: Immediate Cash Flow Part of the Duffee Puzzle.

This table evaluates the abilities of three overlaying theoretical models to fit Facts (a)–(h) and the Duffee Puzzle. These facts are established in Section 2. **Empirical Moments:** Column "Data" presents three empirical benchmarks of each fact: (1) unconditional moments using data during non-recessions ($I_{NBER} = 0$) and (2) recessions ($I_{NBER} = 1$), and (3) a regression coefficient of the DGP-implied conditional moments on the NBER recession indicator. Bootstrapped and OLS standard errors are shown in parentheses under Column "SE"; standard errors for Facts (c) and (e) are obtained using Delta's method. Significance of the equality test between the two unconditional moments are indicated next to the non-recession moment; significance of the regression coefficient of the conditional moments is also shown; $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. **Model moments:** The counterparts using the simulated datasets of the three theoretical models are shown under Columns "M(1)", "M(2)" and "M(3)". The models are solved numerically using the "series method" introduced in Wachter (2005), and simulated for 100,000 months; see details on calibration in Section 4.3.1. All model-implied moments in this paper are calculated using the second half of the simulated dataset, i.e., 50,001-100,000. The algorithm for identifying recession periods is described in Appendix F. **Symbols:** σ, volatility; C, covariance; ρ, correlation; b, sensitivity. Bold (italic) values indicate that the simulation moment point estimates are within a 95% (99%) confidence interval of the empirical moments.

		Data	SE	M(1) Adapted Campbell& Cochrane,1999	M(2) Adapted Bekaert& Engstrom,2017	M(3) This Paper
	s as State Variable	-	-	Yes	Yes	Yes
	n as State Variable	-	-	No	Yes	Yes
	b as State Variable	-	-	No	No	Yes
(a).	$\sigma(\Delta c)$ $(I_{rece.} = 0)$	0.0031*	(0.0001)	**0.0032**	**0.0032**	**0.0032**
	$\sigma(\Delta c)$ $(I_{rece.} = 1)$	0.0036	(0.0002)	0.0032	**0.0035**	**0.0035**
	$\sigma_t(\Delta c_{t+1}) \sim I_{rece.,t}$	4.28E-04***	(2.68E-05)	-	2.47E-04	2.47E-04
(b).	$\sigma(\Delta d)$ $(I_{rece.} = 0)$	0.0118***	(0.0005)	**0.01158**	**0.01173**	**0.01174**
	$\sigma(\Delta d)$ $(I_{rece.} = 1)$	0.0092	(0.0008)	0.01177	0.01214	0.01218
	$\sigma_t(\Delta d_{t+1}) \sim I_{rece.,t}$	-4.35E-06*	(2.59E-06)	-	-	**-4.00E-06**
(c).	$C(\Delta d, \Delta c)$ $(I_{rece.} = 0)$ $(\times 10^5)$	0.1110	(0.1424)	**0.0818**	**0.0770**	**0.0809**
	$C(\Delta d, \Delta c)$ $(I_{rece.} = 1)$ $(\times 10^5)$	-0.0044	(0.1286)	**0.0843**	**0.0868**	**0.0232**
	$C_t(\Delta d_{t+1}, \Delta c_{t+1}) \sim I_{rece.,t}$ $(\times 10^5)$	-0.1033***	(0.0159)	-	-	**-0.1242**
(d).	$\rho(\Delta d, \Delta c)$ $(I_{rece.} = 0)$	0.0303	(0.0388)	**0.0223**	**0.0208**	**0.0218**
	$\rho(\Delta d, \Delta c)$ $(I_{rece.} = 1)$	-0.0013	(0.0388)	**0.0223**	**0.0203**	**0.0054**
	$\rho_t(\Delta d_{t+1}, \Delta c_{t+1}) \sim I_{rece.,t}$	-0.0282***	(0.0042)	-	-0.0012	**-0.0348**
(e).	$b(\Delta d, \Delta c)$ $(I_{rece.} = 0)$	0.1155	(0.1482)	**0.0817**	**0.0770**	**0.0810**
	$b(\Delta d, \Delta c)$ $(I_{rece.} = 1)$	-0.0034	(0.1003)	**0.0815**	**0.0700**	**0.0187**
	$b_t(\Delta d_{t+1}, \Delta c_{t+1}) \sim I_{rece.,t}$	-0.1041***	(0.0155)	-	-0.0075	**-0.1319**
(f).	$\sigma(r^m - \Delta d)$ $(I_{rece.} = 0)$	0.0413***	(0.0020)	0.0146	**0.0423**	**0.0419**
	$\sigma(r^m - \Delta d)$ $(I_{rece.} = 1)$	0.0665	(0.0066)	0.0138	**0.0537**	*0.0509*
(g).	$\sigma(r^m)$ $(I_{rece.} = 0)$	0.0400***	(0.0020)	0.0188	**0.0423**	**0.0420**
	$\sigma(r^m)$ $(I_{rece.} = 1)$	0.0652	(0.0061)	0.0183	**0.0539**	0.0485
(h).	$C(r^m - \Delta d, \Delta c)$ $(I_{rece.} = 0)$ $(\times 10^5)$	1.7436*	(0.4925)	4.0956	3.1658	*2.9983*
	$C(r^m - \Delta d, \Delta c)$ $(I_{rece.} = 1)$ $(\times 10^5)$	3.3938	(0.9146)	**4.1321**	**3.9563**	**3.8239**
(Duffee)	$C(r^m, \Delta c)$ $(I_{rece.} = 0)$ $(\times 10^5)$	1.8546	(0.4767)	4.1775	3.2428	*3.0792*
	$C(r^m, \Delta c)$ $(I_{rece.} = 1)$ $(\times 10^5)$	3.3894	(0.8966)	**4.2165**	**4.0431**	**3.8470**

Table 8: Theoretical Models: Unconditional Moments of the Duffee Puzzle Components.

This table presents 17 unconditional moments from empirical and simulated datasets. Details on data, models, and simulations are described in Table 7. Bootstrapped standard errors are shown in parentheses. Bold (italic) values indicate that the simulation moment point estimates are within a 95% (99%) confidence interval of the empirical moments.

	Data	SE	M(1) Adapted Campbell& Cochrane,1999	M(2) Adapted Bekaert& Engstrom,2017	M(3) This Paper
s as State Variable	-	-	Yes	Yes	Yes
n as State Variable	-	-	No	Yes	Yes
b as State Variable	-	-	No	No	Yes
$E(\Delta c)$	0.0025	(0.0001)	**0.0025**	**0.0025**	**0.0025**
$\sigma(\Delta c)$	0.0032	(0.0001)	**0.0032**	**0.0032**	**0.0032**
$Skew(\Delta c)$	-0.1292	(0.1419)	**-0.2658**	**-0.2707**	**-0.2707**
$xKurt(\Delta c)$	0.7779	(0.3553)	**0.5342**	**0.8354**	**0.8354**
Heteroskedastic Δc Innovations	Yes		No	**Yes**	**Yes**
$E(\Delta d)$	0.0015	(0.0005)	**0.0015**	**0.0015**	**0.0015**
$\sigma(\Delta d)$	0.0116	(0.0005)	**0.0116**	**0.0117**	**0.0118**
$Skew(\Delta d)$	0.2268	(0.2478)	**0.0285**	**0.0117**	**0.0122**
$xKurt(\Delta d)$	2.7560	(0.5656)	-0.0152	-0.0052	-0.0060
Heteroskedastic Δd Innovations	Yes		No	No	**Yes**
$C(\Delta d_{t+1}, \Delta c_{t+1})(\times 10^5)$	0.2140	(0.1341)	**0.0831**	**0.0847**	**0.0849**
$\rho(\Delta d_{t+1}, \Delta c_{t+1})$	0.0569	(0.0343)	**0.0224**	**0.0225**	**0.0225**
$b(\Delta d_{t+1}, \Delta c_{t+1})$	0.2052	(0.1278)	**0.0812**	**0.0824**	**0.0825**
$\sigma(r^m - \Delta d)$	0.0458	(0.0019)	0.0147	**0.0428**	**0.0421**
$\sigma(r^m)$	0.0448	(0.0019)	0.0189	**0.0427**	**0.0422**
$C(r^m - \Delta d, \Delta c)$ $(\times 10^5)$	2.2682	(0.5574)	4.1826	**3.2568**	**3.0889**
$C(r^m, \Delta c)$ $(\times 10^5)$	2.4822	(0.5624)	4.2657	**3.3416**	**3.1738**

Table 9: Theoretical Models: Unconditional Asset Price Statistics ($*$=annualized).

This table presents 10 unconditional moments of asset prices from actual and simulated datasets. Bold (italic) values indicate that the simulation moment point estimates are within a 95% (99%) confidence interval of the empirical moments.

		Data	SE	M(1) Adapted Campbell& Cochrane,1999	M(2) Adapted Bekaert& Engstrom,2017	M(3) This Paper
	s as State Variable	-	-	Yes	Yes	Yes
	n as State Variable	-	-	No	Yes	Yes
	b as State Variable	-	-	No	No	Yes
$*$	$E(r^m - rf), \%$	4.7964	(2.0829)	**3.8751**	**6.2414**	**5.6524**
$*$	$\sigma(r^m - rf), \%$	15.4516	(0.6197)	6.4629	**14.9684**	**14.7234**
	$\exp[E(pd)]$	35.992	(0.5461)	25.8418	17.2668	17.5131
	$\sigma(pd)$	0.3847	(0.0895)	0.1090	**0.2429**	**0.2594**
$*$	$ac(pd)$	0.9236	(0.0557)	**0.9063**	**0.8751**	**0.8757**
	Sharpe Ratio	0.3276	(0.1501)	**0.5992**	**0.4236**	**0.3888**
	Skewness	-0.7932	(0.2592)	0.1515	-0.1816	-0.1453
	xKurtosis	2.6386	(1.2713)	**0.3318**	**0.5579**	**0.4870**
$*$	$E(rf), \%$	1.4854	(0.1525)	*1.1159*	**1.3608**	**1.3608**
$*$	$\sigma(rf), \%$	0.9895	(0.0428)	0.0348	0.0450	0.0450

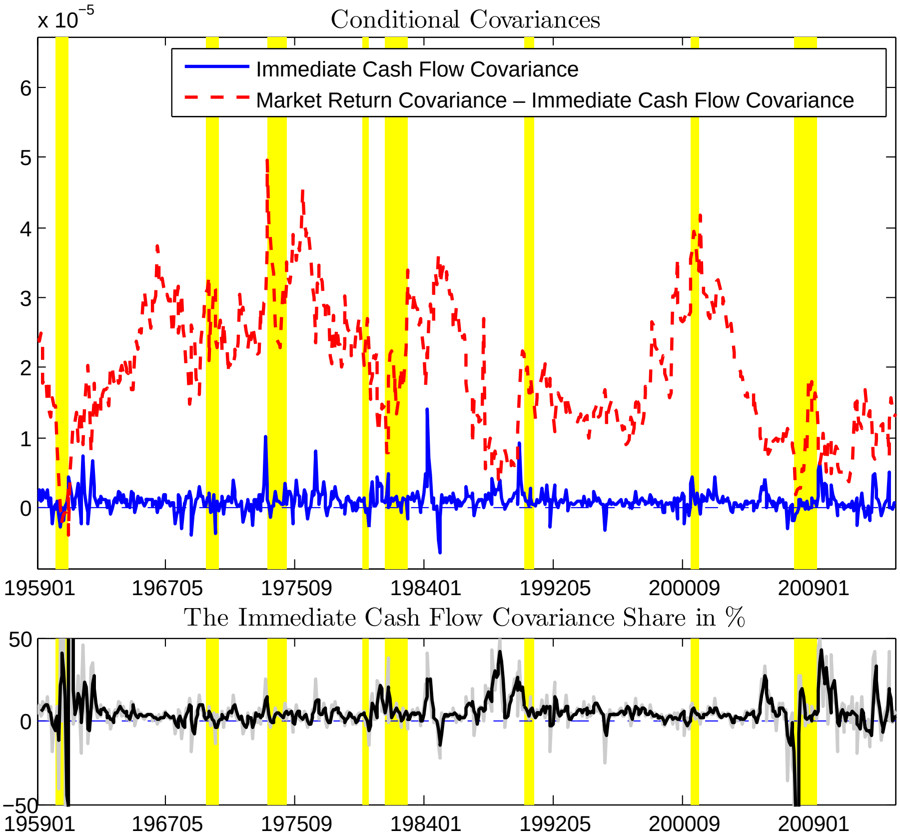


Figure 1: Empirical Model: Time Series of the Duffee Puzzle Components.

The top plot shows the time variation in the immediate cash flow covariance (in solid blue) and the difference between the market return covariance and the cash flow covariance (in dashed red). Both conditional covariances are estimated from a cyclical DCC model. The market return covariance estimates are depicted in Figure 2. The bottom plot depicts the share of the immediate cash flow covariance in the total return covariance, expressed in percentages; the black line is the 3-month moving average for demonstration purpose. The shaded regions are the NBER recession months from the NBER website.

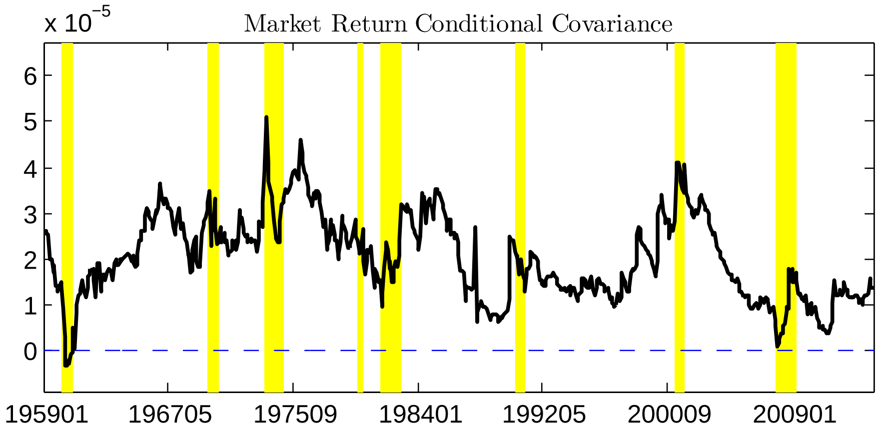


Figure 2: Empirical Model: Time Series of the Conditional Covariance between Market Returns and Consumption Growth.

The shaded regions are the NBER recession months from the NBER website.

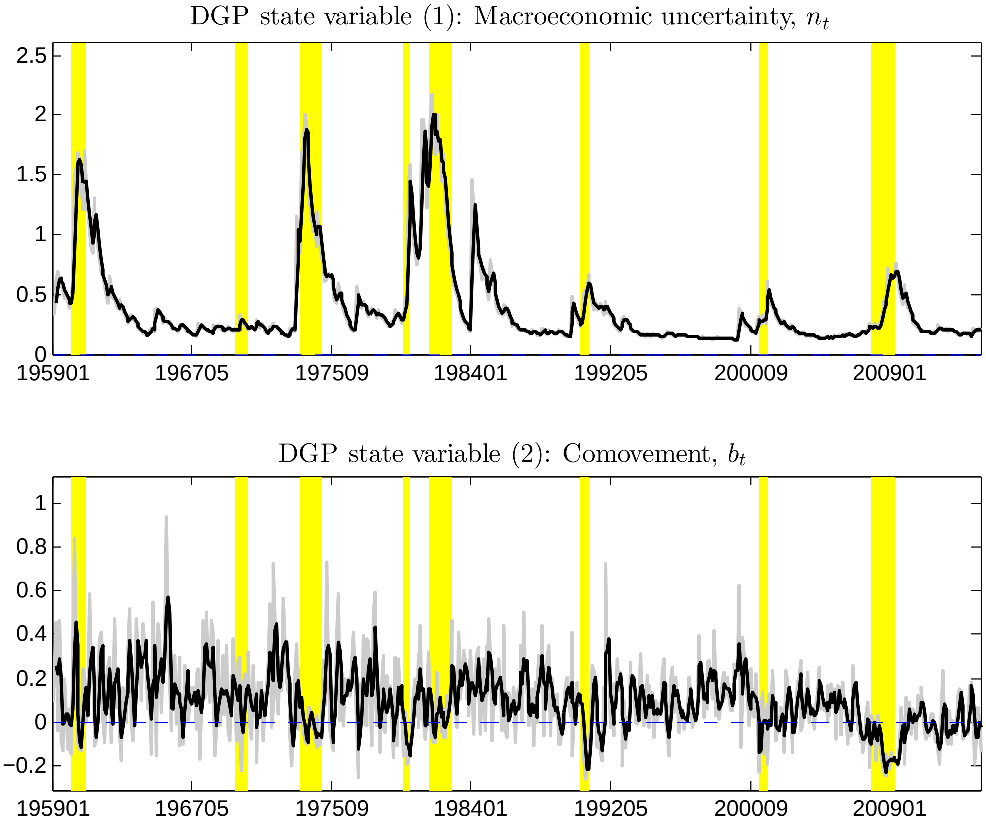


Figure 3: DGP: State Variables.

In both plots, gray line depicts the monthly estimates and the overlaying black line depicts the 3-month moving averages. The countercyclical macroeconomic uncertainty state variable n_t (top plot) is estimated from a filtration-based maximum likelihood estimation methodology developed by Bates (2006), and the procyclical consumption-dividend comovement state variable b_t (bottom plot) is estimated using MLE; detailed estimation procedure is provided in Appendix B; detailed estimation results are shown in Table 4. The monthly n_t (b_t) estimates exhibit a significant correlation of 0.545 (-0.245) with the NBER recession indicator. The shaded regions are the NBER recession months from the NBER website.

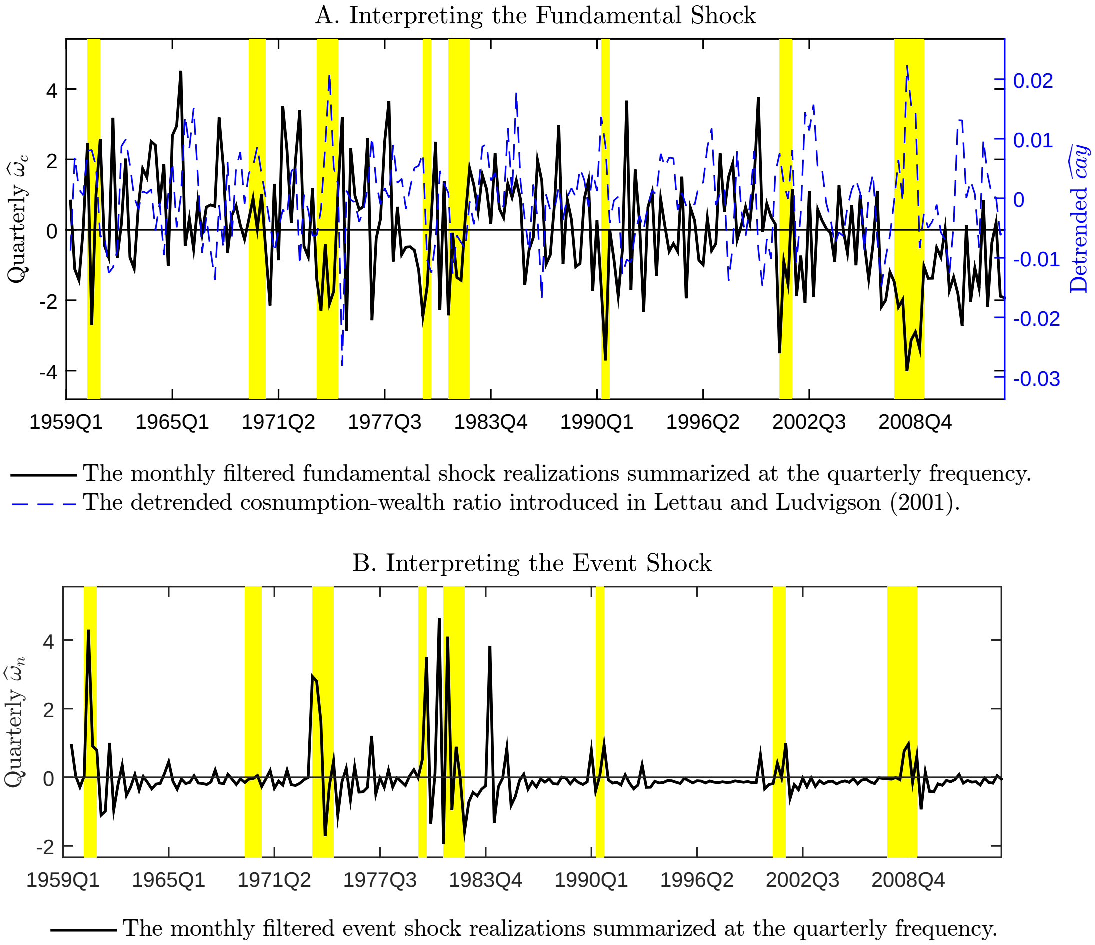


Figure 4: DGP: Economic Interpretations of Consumption Shocks.

The plots provide graphical evidence for the economic interpretations of the fundamental shock and the event shock. Plot A provides a quarter-to-quarter comparison between the fundamental shock (in solid black) and the detrended consumption-wealth ratio (in dashed blue) produced from Lettau and Ludvigson (2001), or $\widehat{cay}$; their correlation is significant and negative (-0.22). Plot B depicts the quarterly event shock realizations; its correlation with the NBER recession indicator is significant and positive (0.25). The shaded regions are the NBER recession quarters from the NBER website.