

# NOT FOR PUBLICATION:

## Online Appendices for “The Time Variation in Risk Appetite and Uncertainty”

### A The state variables

#### A.1 Matrix representation of the state variables

In this section, we show the matrix representation of the system of ten state variables in this economy. The ten state variables, as introduced in Section 3, are as follows,

$$\mathbf{Y}_t = [\theta_t, p_t, n_t, \pi_t, l_t, g_t, \kappa_t, \eta_t, lp_t, qt]'$$

where  $\{p_t, n_t\}$  denote the upside uncertainty factor and the downside uncertainty factor, as latent variables extracted from the system of output growth (i.e., change in log real industrial production index);  $\pi_t$  represents the inflation rate;  $l_t$  represents the log of corporate loss rate;  $g_t$  represents the log change in real earnings;  $\kappa_t$  represents the log consumption-earnings ratio;  $\eta_t$  represents the log dividend payout ratio;  $lp_t$  represents the cash flow uncertainty factor, as the latent variable extracted from the system of corporate loss rate  $l_t$ ;  $qt$  represents the latent risk aversion of the economy. The state variables have the following matrix representation:

$$\mathbf{Y}_{t+1} = \boldsymbol{\mu} + \mathbf{A}\mathbf{Y}_t + \boldsymbol{\Sigma}\boldsymbol{\omega}_{t+1}, \quad (\text{A.1})$$

where  $\boldsymbol{\omega}_{t+1} = [\omega_{p,t+1}, \omega_{n,t+1}, \omega_{\pi,t+1}, \omega_{lp,t+1}, \omega_{ln,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}, \omega_{q,t+1}]$  ( $9 \times 1$ ) is a vector comprised of eight independent shocks in the economy. Among the nine shocks,  $\{\omega_{\pi,t+1}, \omega_{ln,t+1}, \omega_{g,t+1}, \omega_{\kappa,t+1}, \omega_{\eta,t+1}\}$  shocks are homoskedastic. The conditional variance, skewness and higher-order moments of the following four centered gamma shocks— $\omega_{p,t+1}$ ,  $\omega_{n,t+1}$ ,  $\omega_{lp,t+1}$ , and  $\omega_{q,t+1}$ —are assumed to be proportional to  $p_t$ ,  $n_t$ ,  $lp_t$ , and  $qt$  respectively. The underlying distributions for the rest four shocks are assumed to be Gaussian with unit standard deviation.

The constant matrices are defined implicitly,

$$\boldsymbol{\mu} = \begin{bmatrix} (1 - \rho_\theta)\bar{\theta} - m_p\bar{p} - m_n\bar{n} \equiv \theta_0 \\ (1 - \rho_p)\bar{p} \equiv p_0 \\ (1 - \rho_n)\bar{n} \equiv n_0 \\ \pi_0 \\ l_0 \\ g_0 \\ \kappa_0 \\ \eta_0 \\ lp_0 \\ q_0 \end{bmatrix}, \quad (\text{A.2})$$

$$\mathbf{A} = \begin{bmatrix} \rho_\theta & m_p & m_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{\pi\theta} & \rho_{\pi p} & \rho_{\pi n} & \rho_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{lp} & \rho_{ln} & 0 & \rho_{ll} & 0 & 0 & 0 & 0 & 0 \\ \rho_{g\theta} & \rho_{gp} & \rho_{gn} & \rho_{gl} & 0 & \rho_{gg} & 0 & 0 & \rho_{glp} & 0 \\ \rho_{\kappa\theta} & \rho_{\kappa p} & \rho_{\kappa n} & \rho_{\kappa l} & 0 & 0 & \rho_{\kappa\kappa} & 0 & \rho_{\kappa lp} & 0 \\ \rho_{\eta\theta} & \rho_{\eta p} & \rho_{\eta n} & \rho_{\eta l} & 0 & 0 & 0 & \rho_{\eta\eta} & \rho_{\eta lp} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{lp} & 0 \\ 0 & \rho_{qp} & \rho_{qn} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{qq} \end{bmatrix}, \quad (\text{A.3})$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{\theta p} & -\sigma_{\theta n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{pp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{nn} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{\pi p} & \sigma_{\pi n} & \sigma_{\pi\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{lp} & \sigma_{ln} & 0 & \sigma_{llp} & -\sigma_{lln} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{gp} & \sigma_{gn} & 0 & \sigma_{glp} & \sigma_{gln} & \sigma_{gg} & 0 & 0 & 0 & 0 \\ \sigma_{\kappa p} & \sigma_{\kappa n} & 0 & \sigma_{\kappa lp} & \sigma_{\kappa ln} & 0 & \sigma_{\kappa\kappa} & 0 & 0 & 0 \\ \sigma_{\eta p} & \sigma_{\eta n} & 0 & \sigma_{\eta lp} & \sigma_{\eta ln} & 0 & 0 & \sigma_{\eta\eta} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{lp lp} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{qp} & \sigma_{qn} & 0 & 0 & 0 & \sigma_{qg} & \sigma_{q\kappa} & 0 & \sigma_{qq} & 0 \end{bmatrix}. \quad (\text{A.4})$$

Given the moment generating functions (mgf) of gamma and Gaussian distributions, we show that the model is affine,  $\forall \boldsymbol{\nu} \in \mathbb{R}^{10}$ ,

$$\begin{aligned} M_Y(\boldsymbol{\nu}) &:= E_t [\exp(\boldsymbol{\nu}' \mathbf{Y}_{t+1})] = \exp(\boldsymbol{\nu}' \boldsymbol{\mu} + \boldsymbol{\nu}' \mathbf{A} \mathbf{Y}_t) E_t [\exp(\boldsymbol{\nu}' \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+1})] \\ &= \exp \left[ \boldsymbol{\nu}' \mathbf{S}_0 + \frac{1}{2} \boldsymbol{\nu}' \mathbf{S}_1 \boldsymbol{\Sigma}^{other} \mathbf{S}_1' \boldsymbol{\nu} + \mathbf{f}_S(\boldsymbol{\nu}) \mathbf{Y}_t + S_2(\boldsymbol{\nu}) \bar{l}n \right], \end{aligned} \quad (\text{A.5})$$

where  $\mathbf{S}_0 = \boldsymbol{\mu}$  ( $10 \times 1$ ),

$$\mathbf{S}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (\text{A.6})$$

$$\boldsymbol{\Sigma}^{other} = \begin{bmatrix} \sigma_{\pi\pi}^2 & \sigma_{\pi g} & \sigma_{\pi\kappa} & \sigma_{\pi\eta} \\ \sigma_{g\pi} & \sigma_{gg}^2 & \sigma_{g\kappa} & \sigma_{g\eta} \\ \sigma_{\kappa\pi} & \sigma_{\kappa g} & \sigma_{\kappa\kappa}^2 & \sigma_{\kappa\eta} \\ \sigma_{\eta\pi} & \sigma_{\eta g} & \sigma_{\eta\kappa} & \sigma_{\eta\eta}^2 \end{bmatrix} \quad (\text{cov-var matrix of } \{\omega_\pi, \omega_g, \omega_\kappa, \omega_\eta\}), \quad (\text{A.7})$$

$$\mathbf{f}_S(\boldsymbol{\nu}) = \boldsymbol{\nu}' \mathbf{A} + \begin{bmatrix} 0 \\ -\sigma_p(\boldsymbol{\nu}) - \ln(1 - \sigma_p(\boldsymbol{\nu})) \\ -\sigma_n(\boldsymbol{\nu}) - \ln(1 - \sigma_n(\boldsymbol{\nu})) \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma_{lp}(\boldsymbol{\nu}) - \ln(1 - \sigma_{lp}(\boldsymbol{\nu})) \\ -\sigma_q(\boldsymbol{\nu}) - \ln(1 - \sigma_q(\boldsymbol{\nu})) \end{bmatrix}', \quad (\text{A.8})$$

$$S_2(\boldsymbol{\nu}) = -\sigma_{ln}(\boldsymbol{\nu}) - \ln(1 - \sigma_{ln}(\boldsymbol{\nu})), \quad (\text{A.9})$$

$$\sigma_p(\boldsymbol{\nu}) = \boldsymbol{\nu}' \boldsymbol{\Sigma}_{\bullet 1}, \quad (\text{A.10})$$

$$\sigma_n(\boldsymbol{\nu}) = \boldsymbol{\nu}' \boldsymbol{\Sigma}_{\bullet 2}, \quad (\text{A.11})$$

$$\sigma_{lp}(\boldsymbol{\nu}) = \boldsymbol{\nu}' \boldsymbol{\Sigma}_{\bullet 4}, \quad (\text{A.12})$$

$$\sigma_{ln}(\boldsymbol{\nu}) = \boldsymbol{\nu}' \boldsymbol{\Sigma}_{\bullet 5}, \quad (\text{A.13})$$

$$\sigma_q(\boldsymbol{\nu}) = \boldsymbol{\nu}' \boldsymbol{\Sigma}_{\bullet 9}, \quad (\text{A.14})$$

where  $\mathbf{M}_{\bullet j}$  denotes the  $j$ -th column of the matrix  $\mathbf{M}$ .

## A.2 Consumption growth

Consumption growth in this economy is endogenous defined and can be expressed in an affine function:

$$\Delta c_{t+1} = g_{t+1} + \Delta \kappa_{t+1} \quad (\text{A.15})$$

$$= c_0 + \mathbf{c}'_2 \mathbf{Y}_t + \mathbf{c}'_1 \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+1}, \quad (\text{A.16})$$

$$(\text{A.17})$$

where  $c_0 = g_0 + \kappa_0$ ,  $\mathbf{c}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0]'$ , and

$$\mathbf{c}_2 = \begin{bmatrix} \rho_{g\theta} + \rho_{\kappa\theta} \\ \rho_{gp} + \rho_{\kappa p} \\ \rho_{gn} + \rho_{\kappa n} \\ 0 \\ 0 \\ \rho_{gg} \\ \rho_{\kappa\kappa} - 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{A.18})$$

## B Asset Pricing

In this section, we solve the model analytically. First, given consumption growth and changes in risk aversion, the log of real pricing kernel of the economy is derived as an affine function of the state variables. Next, we show that asset prices of claims on cash flows from three different asset markets can be expressed in (quasi) affine equations. The model is solved using the non-arbitrage condition. The goal of this section is to derive the analytical solutions for the expected excess returns, the physical variance of asset returns and the risk-neutral variance of asset returns in closed forms. The implied moments are crucial for the estimation procedure.

### B.1 The real pricing kernel

The log real pricing kernel for this economy is given by,

$$m_{t+1} = \ln(\beta) - \gamma\Delta c_{t+1} + \gamma\Delta q_{t+1} \quad (\text{B.1})$$

$$= m_0 + \mathbf{m}'_2 \mathbf{Y}_t + \mathbf{m}'_1 \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+1}, \quad (\text{B.2})$$

where  $m_0 = \ln(\beta) + \gamma(g_0 - g_0 - \kappa_0)$ ,  $\mathbf{m}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ -\gamma \ -\gamma \ 0 \ 0 \ \gamma]'$ , and

$$\mathbf{m}_2 = \begin{bmatrix} \gamma(-\rho_{g\theta} - \rho_{\kappa\theta}) \\ \gamma(\rho_{qp} - \rho_{gp} - \rho_{\kappa p}) \\ \gamma(\rho_{qn} - \rho_{gn} - \rho_{\kappa n}) \\ 0 \\ 0 \\ -\gamma\rho_{gg} \\ -\gamma(\rho_{\kappa\kappa} - 1) \\ 0 \\ 0 \\ \gamma(\rho_{qq} - 1) \end{bmatrix}. \quad (\text{B.3})$$

As a result, the moment generating function of the real pricing kernel is,  $\forall \nu \in \mathbb{R}$ ,

$$\begin{aligned} E_t [\exp(\nu m_{t+1})] &= \exp[\nu m_0 + \nu \mathbf{m}'_2 \mathbf{Y}_t] \\ &\cdot \exp\{[-\nu\sigma_p(\mathbf{m}_1) - \ln(1 - \nu\sigma_p(\mathbf{m}_1))]p_t + [-\nu\sigma_n(\mathbf{m}_1) - \ln(1 - \nu\sigma_n(\mathbf{m}_1))]n_t\} \\ &\cdot \exp\{[-\nu\sigma_{lp}(\mathbf{m}_1) - \ln(1 - \nu\sigma_{lp}(\mathbf{m}_1))]lp_t + [-\nu\sigma_q(\mathbf{m}_1) - \ln(1 - \nu\sigma_q(\mathbf{m}_1))]q_t\} \\ &\cdot \exp\left\{[-\nu\sigma_{ln}(\mathbf{m}_1) - \ln(1 - \nu\sigma_{ln}(\mathbf{m}_1))]\bar{l}n + \frac{1}{2}\nu^2 [\mathbf{m}'_1 \mathbf{S}_1 \boldsymbol{\Sigma}^{other} \mathbf{S}'_1 \mathbf{m}_1]\right\}, \end{aligned} \quad (\text{B.4})$$

where  $m_0$ ,  $\mathbf{m}_1$ ,  $\mathbf{m}_2$ ,  $\mathbf{S}_1$ , and  $\boldsymbol{\Sigma}^{other}$  are constant matrices defined earlier, and

$$\sigma_p(\mathbf{m}_1) = \mathbf{m}'_1 \boldsymbol{\Sigma}_{\bullet 1}, \quad (\text{B.5})$$

$$\sigma_n(\mathbf{m}_1) = \mathbf{m}'_1 \boldsymbol{\Sigma}_{\bullet 2}, \quad (\text{B.6})$$

$$\sigma_{lp}(\mathbf{m}_1) = \mathbf{m}'_1 \boldsymbol{\Sigma}_{\bullet 4}, \quad (\text{B.7})$$

$$\sigma_{ln}(\mathbf{m}_1) = \mathbf{m}'_1 \boldsymbol{\Sigma}_{\bullet 5}, \quad (\text{B.8})$$

$$\sigma_q(\mathbf{m}_1) = \mathbf{m}'_1 \boldsymbol{\Sigma}_{\bullet 9}. \quad (\text{B.9})$$

Accordingly, the model-implied short rate  $r f_t$  is,

$$r f_t = -\ln\{E_t[\exp(m_{t+1})]\} \quad (\text{B.10})$$

$$= -m_0 - \mathbf{m}'_2 \mathbf{Y}_t \quad (\text{B.11})$$

$$+ [\sigma_p(\mathbf{m}_1) + \ln(1 - \sigma_p(\mathbf{m}_1))] p_t + [\sigma_n(\mathbf{m}_1) + \ln(1 - \sigma_n(\mathbf{m}_1))] n_t \quad (\text{B.12})$$

$$+ [\sigma_{lp}(\mathbf{m}_1) + \ln(1 - \sigma_{lp}(\mathbf{m}_1))] lp_t + [\sigma_q(\mathbf{m}_1) + \ln(1 - \sigma_q(\mathbf{m}_1))] q_t \quad (\text{B.13})$$

$$+ [\sigma_{ln}(\mathbf{m}_1) + \ln(1 - \sigma_{ln}(\mathbf{m}_1))] \bar{ln} - \frac{1}{2} [\mathbf{m}'_1 \mathbf{S}_1 \Sigma^{other} \mathbf{S}'_1 \mathbf{m}_1], \quad (\text{B.14})$$

$$= rf_0 + \mathbf{r} \mathbf{f}'_2 \mathbf{Y}_t. \quad (\text{B.15})$$

To price nominal assets, we define the nominal pricing kernel,  $\tilde{m}_{t+1}$ , which is a simple transformation of the log real pricing kernel,  $m_{t+1}$ ,

$$\tilde{m}_{t+1} = m_{t+1} - \pi_{t+1}, \quad (\text{B.16})$$

$$= \tilde{m}_0 + \tilde{\mathbf{m}}'_2 \mathbf{Y}_t + \tilde{\mathbf{m}}'_1 \Sigma \omega_{t+1}, \quad (\text{B.17})$$

where  $\tilde{m}_0 = m_0 - \pi_0$ ,  $\tilde{\mathbf{m}}_1 = \mathbf{m}_1 - [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]'$ , and

$$\tilde{\mathbf{m}}_2 = \mathbf{m}_2 - \begin{bmatrix} \rho_{\pi\theta} \\ \rho_{\pi p} \\ \rho_{\pi n} \\ \rho_{\pi\pi} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{B.18})$$

The nominal risk free rate  $\tilde{r}f_t$  is defined as  $-\ln\{E_t[\exp(\tilde{m}_{t+1})]\}$ .

## B.2 Valuation ratio

It is a crucial step in this paper to show that asset prices are (quasi) affine functions of the state variables.

**Defaultable Nominal Bonds** In the paper, we assume that a one period nominal bond faces a fractional (logarithmic) loss of  $l_t$ . Given the structures assumed for  $l_t$  and  $\pi_t$  and the model-implied log pricing kernel, the price-coupon ratio of the one-period defaultable bond portfolio is

$$PC_t^1 = E_t[\exp(\tilde{m}_{t+1} - l_{t+1})] \quad (\text{B.19})$$

$$= \exp(b_0^1 + \mathbf{b}_1^{1'} \mathbf{Y}_t), \quad (\text{B.20})$$

where  $b_0^1$  and  $\mathbf{b}_1^{1'}$  are implicitly defined. Consider next a portfolio of multi-period zero-coupon defaultable bonds with a promised terminal payment of  $C$  at period  $(t + N)$ . As for the  $N$ -period bond, the actual payment will be less than or equal to the promised payment, and the ex-post nominal payoff can be expressed as  $\exp(c - l_{t+N})$ . We ignore the possibility of early default or prepayment. Then, the price-coupon ratio of this bond at period  $(t + N - 1)$ , one period before maturity,  $PC_{t+N-1}^1$ , is  $\exp(b_0^1 + \mathbf{b}_1^{1'} \mathbf{Y}_{t+N-1})$ . Given the Euler equation and the law of iterated expectations, it then follows by induction that all earlier dated zero-coupon nominally defaultable corporate bond (maturing in  $N$  period) prices are similarly affine in the state variables, in particular:

$$PC_t^N = E_t[\tilde{M}_{t+1} PC_{t+1}^{N-1}],$$

$$= \exp(b_0^N + \mathbf{b}_1^{N'} \mathbf{Y}_t). \quad (\text{B.21})$$

Therefore, the assumed zero-coupon structure of the payments before maturity implies that the unexpected returns to this portfolio are exactly linearly spanned by the shocks to  $\mathbf{Y}_t$ .

**Equity** It is especially not obvious for equity price-dividend ratio, of which we provide proofs below. First, we rewrite the real dividend growth in a general matrix expression:

$$\Delta d_{t+1} = g_{t+1} + \Delta \eta_{t+1}$$

$$= h_0 + \mathbf{h}'_2 \mathbf{Y}_t + \mathbf{h}'_1 \Sigma \omega_{t+1}, \quad (\text{B.22})$$

where  $h_0 = g_0 + \eta_0$ ,  $\mathbf{h}_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0]'$ , and

$$\mathbf{h}_2 = \begin{bmatrix} \rho_{g\theta} + \rho_{\eta\theta} \\ \rho_{gp} + \rho_{\eta p} \\ \rho_{gn} + \rho_{\eta n} \\ 0 \\ 0 \\ \rho_{gg} \\ 0 \\ \rho_{\eta\eta} - 1 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{B.23})$$

The price-dividend ratio,  $PD_t = E_t \left[ M_{t+1} \left( \frac{P_{t+1} + D_{t+1}}{D_t} \right) \right]$ , can be rewritten as,

$$PD_t = \sum_{n=1}^{\infty} E_t \left[ \exp \left( \sum_{j=1}^n m_{t+j} + \Delta d_{t+j} \right) \right]. \quad (\text{B.24})$$

Let  $F_t^n$  denote the  $n$ -th term in the summation:

$$F_t^n = E_t \left[ \exp \left( \sum_{j=1}^n m_{t+j} + \Delta d_{t+j} \right) \right], \quad (\text{B.25})$$

and  $F_t^n D_t$  is the price of zero-coupon equity that matures in  $n$  periods.

To show that equity price is an approximate affine function of the state variables, we first prove that  $F_t^n (\forall n \geq 1)$  is exactly affine using induction. First, when  $n = 1$ ,

$$\begin{aligned} F_t^1 &= E_t [\exp (m_{t+1} + \Delta d_{t+1})] \\ &= E_t \{ \exp [(m_0 + h_0) + (\mathbf{m}'_2 + \mathbf{h}'_2) \mathbf{Y}_t + (\mathbf{m}'_1 + \mathbf{h}'_1) \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+1}] \} \\ &= \exp [(m_0 + h_0) + (\mathbf{m}'_2 + \mathbf{h}'_2) \mathbf{Y}_t] \\ &\cdot \exp \{ [-\sigma_p(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_p(\mathbf{m}_1 + \mathbf{h}_1))] p_t + [-\sigma_n(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_n(\mathbf{m}_1 + \mathbf{h}_1))] n_t \} \\ &\cdot \exp \{ [-\sigma_{lp}(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_{lp}(\mathbf{m}_1 + \mathbf{h}_1))] lp_t + [-\sigma_q(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_q(\mathbf{m}_1 + \mathbf{h}_1))] qt_t \} \\ &\cdot \exp \left\{ [-\sigma_{ln}(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_{ln}(\mathbf{m}_1 + \mathbf{h}_1))] \bar{ln} + \frac{1}{2} [(\mathbf{m}'_1 + \mathbf{h}'_1) \mathbf{S}_1 \boldsymbol{\Sigma}^{other} \mathbf{S}'_1 (\mathbf{m}_1 + \mathbf{h}_1)] \right\} \\ &= \exp (e_0^1 + \mathbf{e}_1^1 \mathbf{Y}_t), \end{aligned} \quad (\text{B.26})$$

where  $m_0$ ,  $\mathbf{m}_1$ ,  $\mathbf{m}_2$ ,  $h_0$ ,  $\mathbf{h}_1$ ,  $\mathbf{h}_2$ ,  $\mathbf{S}_1$ , and  $\boldsymbol{\Sigma}^{other}$  are constant matrices defined earlier, and

$$\sigma_p(\mathbf{m}_1 + \mathbf{h}_1) = (\mathbf{m}'_1 + \mathbf{h}'_1) \boldsymbol{\Sigma}_{\bullet 1}, \quad (\text{B.27})$$

$$\sigma_n(\mathbf{m}_1 + \mathbf{h}_1) = (\mathbf{m}'_1 + \mathbf{h}'_1) \boldsymbol{\Sigma}_{\bullet 2}, \quad (\text{B.28})$$

$$\sigma_{lp}(\mathbf{m}_1 + \mathbf{h}_1) = (\mathbf{m}'_1 + \mathbf{h}'_1) \boldsymbol{\Sigma}_{\bullet 4}, \quad (\text{B.29})$$

$$\sigma_{ln}(\mathbf{m}_1 + \mathbf{h}_1) = (\mathbf{m}'_1 + \mathbf{h}'_1) \boldsymbol{\Sigma}_{\bullet 5}, \quad (\text{B.30})$$

$$\sigma_q(\mathbf{m}_1 + \mathbf{h}_1) = (\mathbf{m}'_1 + \mathbf{h}'_1) \boldsymbol{\Sigma}_{\bullet 9}, \quad (\text{B.31})$$

and  $e_0^1 = m_0 + h_0 + [-\sigma_{ln}(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_{ln}(\mathbf{m}_1 + \mathbf{h}_1))] \bar{ln} + \frac{1}{2} [(\mathbf{m}'_1 + \mathbf{h}'_1) \mathbf{S}_1 \boldsymbol{\Sigma}^{other} \mathbf{S}'_1 (\mathbf{m}_1 + \mathbf{h}_1)]$ , and

$$\mathbf{e}_1^1 = \mathbf{m}_2 + \mathbf{h}_2 + \begin{bmatrix} 0 \\ -\sigma_p(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_p(\mathbf{m}_1 + \mathbf{h}_1)) \\ -\sigma_n(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_n(\mathbf{m}_1 + \mathbf{h}_1)) \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sigma_{lp}(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_{lp}(\mathbf{m}_1 + \mathbf{h}_1)) \\ -\sigma_q(\mathbf{m}_1 + \mathbf{h}_1) - \ln(1 - \sigma_q(\mathbf{m}_1 + \mathbf{h}_1)) \end{bmatrix}. \quad (\text{B.32})$$

Now, suppose that the  $(n-1)$ -th term  $F_t^{n-1} = \exp(e_0^{n-1} + \mathbf{e}_1^{n-1'} \mathbf{Y}_t)$ , then

$$\begin{aligned}
F_t^n &= E_t \left[ \exp \left( \sum_{j=1}^n m_{t+j} + \Delta d_{t+j} \right) \right] \\
&= E_t \left\{ E_{t+1} \left[ \exp(m_{t+1} + \Delta d_{t+1}) \exp \left( \sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1} \right) \right] \right\} \\
&= E_t \left\{ \exp(m_{t+1} + \Delta d_{t+1}) \underbrace{E_{t+1} \left[ \exp \left( \sum_{j=1}^{n-1} m_{t+j+1} + \Delta d_{t+j+1} \right) \right]}_{F_{t+1}^{n-1}} \right\} \\
&= E_t \left[ \exp(m_{t+1} + \Delta d_{t+1}) \exp(e_0^{n-1} + \mathbf{e}_1^{n-1'} \mathbf{Y}_{t+1}) \right] \\
&= \exp(e_0^n + \mathbf{e}_1^{n'} \mathbf{Y}_t),
\end{aligned} \tag{B.33}$$

where  $e_0^n$  and  $\mathbf{e}_1^{n'}$  are defined implicitly.

Hence, the price-dividend ratio is approximately affine:

$$\begin{aligned}
PD_t &= \sum_{n=1}^{\infty} E_t \left[ \exp \left( \sum_{j=1}^n m_{t+j} + \Delta d_{t+j} \right) \right] \\
&= \sum_{n=1}^{\infty} F_t^n \\
&= \sum_{n=1}^{\infty} \exp(e_0^n + \mathbf{e}_1^{n'} \mathbf{Y}_t).
\end{aligned} \tag{B.34}$$

■

### B.3 Log asset returns

#### Log return of zero-coupon nominally defaultable corporate bonds maturing at $t+N$

Given the exact exponential affine expression of the valuation ratio of this asset (see derivations above), the log return can be derived an approximate linear closed form:

$$\begin{aligned}
\tilde{r}_{t+1}^{cb,N} &= \ln \left( \frac{PC_{t+1}^{N-1} + 1}{PC_t^N} \right) \ln \left( \frac{C}{\bar{C}} \right) \ln(\Pi_{t+1}) \\
&= \pi_{t+1} + \ln \left[ \frac{1 + \exp(b_0^{N-1} + \mathbf{b}_1^{N-1'} \mathbf{Y}_{t+1})}{\exp(b_0^N + \mathbf{b}_1^{N'} \mathbf{Y}_t)} \right] \\
&\approx \pi_{t+1} + \text{const.} + \frac{\exp(b_0^{N-1} + \mathbf{b}_1^{N-1'} \bar{\mathbf{Y}} \mathbf{b}_1^{N-1'})}{\frac{1 + \exp(b_0^{N-1} + \mathbf{b}_1^{N-1'} \bar{\mathbf{Y}})}{\exp(b_0^N + \mathbf{b}_1^{N'} \bar{\mathbf{Y}})}} \mathbf{Y}_{t+1} - \mathbf{b}_1^{N'} \mathbf{Y}_t \\
&= \tilde{\xi}_0^{cb} + \tilde{\xi}_1^{cb'} \mathbf{Y}_t + \tilde{\mathbf{r}}^{cb'} \boldsymbol{\Sigma} \boldsymbol{\omega}_{t+1},
\end{aligned} \tag{B.35}$$

where  $\tilde{r}_{t+1}^{cb}$  is the log *nominal* return of corporate bond from  $t$  to  $t+1$ ,  $\tilde{\xi}_0^{cb}$  is constant,  $\tilde{\xi}_1^{cb}$  is a vector of state vector coefficients, and  $\tilde{\mathbf{r}}^{cb}$  is a vector of shock coefficients. Thus, this step involves linear approximation.

**Log nominal equity return** We apply first-order Taylor approximations to the log nominal equity return, and obtain a linear system,

$$\begin{aligned}
\tilde{r}_{t+1}^{eq} &= \ln \left( \frac{P_{t+1} + D_{t+1}}{P_t} \Pi_{t+1} \right) \\
&= \ln \left( \frac{PD_{t+1} + 1}{PD_t} \right) \ln \left( \frac{D_{t+1}}{D_t} \right) \ln(\Pi_{t+1}) \\
&= \Delta d_{t+1} + \pi_{t+1} + \ln \left[ \frac{1 + \sum_{n=1}^{\infty} \exp(e_0^n + \mathbf{e}_1^{n'} \mathbf{Y}_{t+1})}{\sum_{n=1}^{\infty} \exp(e_0^n + \mathbf{e}_1^{n'} \mathbf{Y}_t)} \right]
\end{aligned}$$

$$\begin{aligned}
&\approx \Delta d_{t+1} + \pi_{t+1} + \text{const.} + \frac{\sum_{n=1}^{\infty} \exp(e_0^n + e_1^{n'} \bar{Y}) e_1^{n'}}{1 + \frac{\sum_{n=1}^{\infty} \exp(e_0^n + e_1^{n'} \bar{Y})}{\sum_{n=1}^{\infty} \exp(e_0^n + e_1^{n'} \bar{Y})}} Y_{t+1} - \frac{\sum_{n=1}^{\infty} \exp(e_0^n + e_1^{n'} \bar{Y}) e_1^{n'}}{\sum_{n=1}^{\infty} \exp(e_0^n + e_1^{n'} \bar{Y})} Y_t \\
&= \tilde{\xi}_0^{eq} + \tilde{\xi}_1^{eq'} Y_t + \tilde{r}^{eq'} \Sigma \omega_{t+1},
\end{aligned} \tag{B.36}$$

where  $\tilde{r}_{t+1}^{eq}$  is the log *nominal* return of equity from  $t$  to  $t+1$ ,  $\tilde{\xi}_0^{eq}$  is constant,  $\tilde{\xi}_1^{eq'}$  is a vector of state vector coefficients, and  $\tilde{r}^{eq'}$  is a vector of shock coefficients. Thus, this step involves linear approximation.

**General expression** To acknowledge the errors that are potentially caused by the linear approximations (the Taylor approximation in log price-dividend ratio in the return equation), we write down the return innovations for asset  $i$  with an idiosyncratic shock:

$$\tilde{r}_{t+1}^i - E_t(\tilde{r}_{t+1}^i) = \tilde{r}^{i'} \Sigma \omega_{t+1} + \varepsilon_{t+1}^i, \tag{B.37}$$

where  $E_t(\tilde{r}_{t+1}^i)$  is the expected return,  $\tilde{r}^i$  ( $10 \times 1$ ) is the asset  $i$  return loadings on selected state variable innovations (the choice of which depends on the asset classes), and  $\varepsilon_{t+1}^i$  is the Gaussian noise uncorrelated with the state variable shocks but may be cross-correlated (with other asset-specific shocks). The Gaussian shock  $\varepsilon_{t+1}^i$  has an unconditional variance  $\sigma_i^2$ .

## B.4 Model-implied moments

In this section, we derive three model-implied asset conditional moments—expected excess returns, physical and risk-neutral conditional variances of nominal asset returns. The moments are crucial in creating the moment conditions during the third step of model estimation.

### B.4.1 One-period expected excess return

We impose the no-arbitrage condition,  $1 = E_t[\exp(\tilde{m}_{t+1} + \tilde{r}_{t+1}^i)]$  ( $\forall i \in \{\text{equity, treasury bond, corporate bond}\}$ ), and obtain the expected excess returns. Expand the law of one price (LOOP) equation:

$$\begin{aligned}
1 &= E_t[\exp(\tilde{m}_{t+1} + \tilde{r}_{t+1}^i)] \\
&= \exp\left[E_t(\tilde{m}_{t+1}) + E_t(\tilde{r}_{t+1}^i)\right] \\
&\cdot \exp\left\{\left[-\sigma_p(\tilde{m}_1 + \tilde{r}^i) - \ln\left(1 - \sigma_p(\tilde{m}_1 + \tilde{r}^i)\right)\right] p_t + \left[-\sigma_n(\tilde{m}_1 + \tilde{r}^i) - \ln\left(1 - \sigma_n(\tilde{m}_1 + \tilde{r}^i)\right)\right] n_t\right\} \\
&\cdot \exp\left\{\left[-\sigma_{lp}(\tilde{m}_1 + \tilde{r}^i) - \ln\left(1 - \sigma_{lp}(\tilde{m}_1 + \tilde{r}^i)\right)\right] lp_t + \left[-\sigma_q(\tilde{m}_1 + \tilde{r}^i) - \ln\left(1 - \sigma_q(\tilde{m}_1 + \tilde{r}^i)\right)\right] qt\right\} \\
&\cdot \exp\left\{\left[-\sigma_{ln}(\tilde{m}_1 + \tilde{r}^i) - \ln\left(1 - \sigma_{ln}(\tilde{m}_1 + \tilde{r}^i)\right)\right] ln\right\} \\
&\cdot \exp\left\{\frac{1}{2} \left[(\tilde{m}_1' + \tilde{r}^{i'}) \mathbf{S}_1 \Sigma^{other} \mathbf{S}_1' (\tilde{m}_1 + \tilde{r}^i) + \sigma_i^2\right]\right\},
\end{aligned} \tag{B.38}$$

where  $\tilde{m}_1$ ,  $\tilde{r}^i$ ,  $\sigma_i$ ,  $\mathbf{S}_1$ , and  $\Sigma^{other}$  are constant matrices defined earlier, and

$$\begin{aligned}
\sigma_p(\tilde{m}_1 + \tilde{r}^i) &= (\tilde{m}_1' + \tilde{r}^{i'}) \Sigma_{\bullet 1}, \\
\sigma_n(\tilde{m}_1 + \tilde{r}^i) &= (\tilde{m}_1' + \tilde{r}^{i'}) \Sigma_{\bullet 2}, \\
\sigma_{lp}(\tilde{m}_1 + \tilde{r}^i) &= (\tilde{m}_1' + \tilde{r}^{i'}) \Sigma_{\bullet 4}, \\
\sigma_{ln}(\tilde{m}_1 + \tilde{r}^i) &= (\tilde{m}_1' + \tilde{r}^{i'}) \Sigma_{\bullet 5}, \\
\sigma_q(\tilde{m}_1 + \tilde{r}^i) &= (\tilde{m}_1' + \tilde{r}^{i'}) \Sigma_{\bullet 9}.
\end{aligned} \tag{B.39}$$

Given the nominal risk free rate derived earlier using real pricing kernel and inflation, the nominal excess return is,

$$\begin{aligned}
E_t(\tilde{r}_{t+1}^i) - \tilde{r}_t &= \left\{ \sigma_p(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_p(\tilde{m}_1 + \tilde{r}^i)}{1 - \sigma_p(\tilde{m}_1)} \right] \right\} p_t \\
&+ \left\{ \sigma_n(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_n(\tilde{m}_1 + \tilde{r}^i)}{1 - \sigma_n(\tilde{m}_1)} \right] \right\} n_t \\
&+ \left\{ \sigma_{lp}(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_{lp}(\tilde{m}_1 + \tilde{r}^i)}{1 - \sigma_{lp}(\tilde{m}_1)} \right] \right\} lp_t \\
&+ \left\{ \sigma_q(\tilde{r}^i) + \ln \left[ \frac{1 - \sigma_q(\tilde{m}_1 + \tilde{r}^i)}{1 - \sigma_q(\tilde{m}_1)} \right] \right\} qt
\end{aligned}$$

$$+ \underbrace{\left\{ \sigma_{ln}(\tilde{\mathbf{r}}^i) + \ln \left[ \frac{1 - \sigma_{ln}(\tilde{\mathbf{m}}_1 + \tilde{\mathbf{r}}^i)}{1 - \sigma_{ln}(\tilde{\mathbf{m}}_1)} \right] \right\}}_{\equiv C(RP^i)} \bar{ln} - \tilde{\mathbf{m}}_1' \mathbf{S}_1 \Sigma^{other} \mathbf{S}_1' \tilde{\mathbf{r}}^i - \frac{1}{2} \left[ \tilde{\mathbf{r}}^{i'} \mathbf{S}_1 \Sigma^{other} \mathbf{S}_1' \tilde{\mathbf{r}}^i + \sigma_i^2 \right] \quad (\text{B.40})$$

where

$$\sigma_p(\tilde{\mathbf{r}}^i) = \tilde{\mathbf{r}}^{i'} \Sigma_{\bullet 1}, \quad (\text{B.41})$$

$$\sigma_n(\tilde{\mathbf{r}}^i) = \tilde{\mathbf{r}}^{i'} \Sigma_{\bullet 2}, \quad (\text{B.42})$$

$$\sigma_{lp}(\tilde{\mathbf{r}}^i) = \tilde{\mathbf{r}}^{i'} \Sigma_{\bullet 4}, \quad (\text{B.43})$$

$$\sigma_{ln}(\tilde{\mathbf{r}}^i) = \tilde{\mathbf{r}}^{i'} \Sigma_{\bullet 5}, \quad (\text{B.44})$$

$$\sigma_q(\tilde{\mathbf{r}}^i) = \tilde{\mathbf{r}}^{i'} \Sigma_{\bullet 9}, \quad (\text{B.45})$$

$$\sigma_p(\tilde{\mathbf{m}}_1 + \tilde{\mathbf{r}}^i) = (\tilde{\mathbf{m}}_1' + \tilde{\mathbf{r}}^{i'}) \Sigma_{\bullet 1}, \quad (\text{B.46})$$

$$\sigma_n(\tilde{\mathbf{m}}_1 + \tilde{\mathbf{r}}^i) = (\tilde{\mathbf{m}}_1' + \tilde{\mathbf{r}}^{i'}) \Sigma_{\bullet 2}, \quad (\text{B.47})$$

$$\sigma_{lp}(\tilde{\mathbf{m}}_1 + \tilde{\mathbf{r}}^i) = (\tilde{\mathbf{m}}_1' + \tilde{\mathbf{r}}^{i'}) \Sigma_{\bullet 4}, \quad (\text{B.48})$$

$$\sigma_{ln}(\tilde{\mathbf{m}}_1 + \tilde{\mathbf{r}}^i) = (\tilde{\mathbf{m}}_1' + \tilde{\mathbf{r}}^{i'}) \Sigma_{\bullet 5}, \quad (\text{B.49})$$

$$\sigma_q(\tilde{\mathbf{m}}_1 + \tilde{\mathbf{r}}^i) = (\tilde{\mathbf{m}}_1' + \tilde{\mathbf{r}}^{i'}) \Sigma_{\bullet 9}. \quad (\text{B.50})$$

### B.4.2 One-period physical conditional return variance

The physical variance is easily obtained given the loadings:

$$\begin{aligned} VAR_t(\tilde{r}_{t+1}^i) &= \left( \sigma_p(\tilde{\mathbf{r}}^i) \right)^2 p_t + \left( \sigma_n(\tilde{\mathbf{r}}^i) \right)^2 n_t + \left( \sigma_{lp}(\tilde{\mathbf{r}}^i) \right)^2 lp_t + \left( \sigma_q(\tilde{\mathbf{r}}^i) \right)^2 qt \\ &\quad + \underbrace{\left( \sigma_{ln}(\tilde{\mathbf{r}}^i) \right)^2 \bar{ln} + \tilde{\mathbf{r}}^{i'} \mathbf{S}_1 \Sigma^{other} \mathbf{S}_1' \tilde{\mathbf{r}}^i + \sigma_i^2}_{\equiv C(P^i)}. \end{aligned} \quad (\text{B.51})$$

### B.4.3 One-period risk-neutral conditional return variance

To obtain the risk-neutral variance of the asset returns, we use the moment generating function under the risk-neutral measure:

$$\begin{aligned} mgf_t^Q(\tilde{r}_{t+1}^i; \nu) &= \frac{E_t \left[ \exp(\tilde{m}_{t+1} + \nu \tilde{r}_{t+1}^i) \right]}{E_t \left[ \exp(\tilde{m}_{t+1}) \right]} \\ &= \exp \left\{ E_t(\tilde{m}_{t+1}) + \nu E_t(\tilde{r}_{t+1}^i) \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_p(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) - \ln \left( 1 - \sigma_p(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) \right) \right] p_t \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_n(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) - \ln \left( 1 - \sigma_n(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) \right) \right] n_t \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_{lp}(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) - \ln \left( 1 - \sigma_{lp}(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) \right) \right] lp_t \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_q(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) - \ln \left( 1 - \sigma_q(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) \right) \right] qt \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_{ln}(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) - \ln \left( 1 - \sigma_{ln}(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) \right) \right] \bar{ln} \right\} \\ &\quad \cdot \exp \left\{ \frac{1}{2} \left[ (\tilde{\mathbf{m}}_1' + \nu \tilde{\mathbf{r}}^{i'}) \mathbf{S}_1 \Sigma^{other} \mathbf{S}_1' (\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i) + \nu^2 \sigma_i^2 \right] \right\} \\ &\quad / \exp \{ E_t(\tilde{m}_{t+1}) \} \\ &\quad / \exp \{ [-\sigma_p(\tilde{\mathbf{m}}_1) - \ln(1 - \sigma_p(\tilde{\mathbf{m}}_1))] p_t + [-\sigma_n(\tilde{\mathbf{m}}_1) - \ln(1 - \sigma_n(\tilde{\mathbf{m}}_1))] n_t \} \\ &\quad / \exp \{ [-\sigma_{lp}(\tilde{\mathbf{m}}_1) - \ln(1 - \sigma_{lp}(\tilde{\mathbf{m}}_1))] lp_t + [-\sigma_q(\tilde{\mathbf{m}}_1) - \ln(1 - \sigma_q(\tilde{\mathbf{m}}_1))] qt \} \\ &\quad / \exp \left\{ [-\sigma_{ln}(\tilde{\mathbf{m}}_1) - \ln(1 - \sigma_{ln}(\tilde{\mathbf{m}}_1))] \bar{ln} + \frac{1}{2} \left[ \tilde{\mathbf{m}}_1' \mathbf{S}_1 \Sigma^{other} \mathbf{S}_1' \tilde{\mathbf{m}}_1 \right] \right\} \\ &= \exp \left\{ \nu E_t(\tilde{r}_{t+1}^i) \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_p(\nu \tilde{\mathbf{r}}^i) - \ln \left( \frac{1 - \sigma_p(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i)}{1 - \sigma_p(\tilde{\mathbf{m}}_1)} \right) \right] p_t \right\} \\ &\quad \cdot \exp \left\{ \left[ -\sigma_n(\nu \tilde{\mathbf{r}}^i) - \ln \left( \frac{1 - \sigma_n(\tilde{\mathbf{m}}_1 + \nu \tilde{\mathbf{r}}^i)}{1 - \sigma_n(\tilde{\mathbf{m}}_1)} \right) \right] n_t \right\} \end{aligned}$$

$$\begin{aligned}
& \cdot \exp \left\{ \left[ -\sigma_{lp}(\nu \tilde{r}^i) - \ln \left( \frac{1 - \sigma_{lp}(\tilde{\mathbf{m}}_1 + \nu \tilde{r}^i)}{1 - \sigma_{lp}(\tilde{\mathbf{m}}_1)} \right) \right] lp_t \right\} \\
& \cdot \exp \left\{ \left[ -\sigma_q(\nu \tilde{r}^i) - \ln \left( \frac{1 - \sigma_q(\tilde{\mathbf{m}}_1 + \nu \tilde{r}^i)}{1 - \sigma_q(\tilde{\mathbf{m}}_1)} \right) \right] qt \right\} \\
& \cdot A(\nu),
\end{aligned} \tag{B.52}$$

where

$$\begin{aligned}
A(\nu) &= \exp \left\{ \left[ -\sigma_{ln}(\nu \tilde{r}^i) - \ln \left( \frac{1 - \sigma_{ln}(\tilde{\mathbf{m}}_1 + \nu \tilde{r}^i)}{1 - \sigma_{ln}(\tilde{\mathbf{m}}_1)} \right) \right] \overline{ln} \right\} \\
&+ \exp \left\{ \frac{1}{2} \left[ (\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) \mathbf{S}_1 \Sigma^{other} \mathbf{S}'_1 (\tilde{\mathbf{m}}_1 + \nu \tilde{r}^i) - \tilde{\mathbf{m}}'_1 \mathbf{S}_1 \Sigma^{other} \mathbf{S}'_1 \tilde{\mathbf{m}}_1 + \nu^2 \sigma_i^2 \right] \right\}
\end{aligned} \tag{B.53}$$

, and

$$\sigma_p(\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) = (\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) \Sigma_{\bullet 1}, \tag{B.54}$$

$$\sigma_n(\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) = (\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) \Sigma_{\bullet 2}, \tag{B.55}$$

$$\sigma_{lp}(\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) = (\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) \Sigma_{\bullet 4}, \tag{B.56}$$

$$\sigma_{ln}(\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) = (\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) \Sigma_{\bullet 5}, \tag{B.57}$$

$$\sigma_q(\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) = (\tilde{\mathbf{m}}'_1 + \nu \tilde{r}^{i'}) \Sigma_{\bullet 9}. \tag{B.58}$$

The first-order moment is the first-order derivate at  $\nu = 0$ :

$$\begin{aligned}
E_t^Q(\tilde{r}_{t+1}^i) &= \frac{\partial mgf_t^Q(\tilde{r}_{t+1}^i; \nu)}{\partial \nu} \Big|_{\nu=0} \\
&= E_t(\tilde{r}_{t+1}^i) + \frac{\sigma_p(\tilde{\mathbf{m}}_1) \sigma_p(\tilde{r}^i)}{1 - \sigma_p(\tilde{\mathbf{m}}_1)} p_t + \frac{\sigma_n(\tilde{\mathbf{m}}_1) \sigma_n(\tilde{r}^i)}{1 - \sigma_n(\tilde{\mathbf{m}}_1)} n_t + \frac{\sigma_{lp}(\tilde{\mathbf{m}}_1) \sigma_{lp}(\tilde{r}^i)}{1 - \sigma_{lp}(\tilde{\mathbf{m}}_1)} lp_t + \frac{\sigma_q(\tilde{\mathbf{m}}_1) \sigma_q(\tilde{r}^i)}{1 - \sigma_q(\tilde{\mathbf{m}}_1)} qt \\
&+ \frac{\sigma_{ln}(\tilde{\mathbf{m}}_1) \sigma_{ln}(\tilde{r}^i)}{1 - \sigma_{ln}(\tilde{\mathbf{m}}_1)} \overline{ln} + \tilde{\mathbf{m}}'_1 \mathbf{S}_1 \Sigma^{other} \mathbf{S}'_1 \tilde{r}^i.
\end{aligned} \tag{B.59}$$

Note the similarity between  $E_t(\tilde{r}_{t+1}^i) - E_t^Q(\tilde{r}_{t+1}^i)$  from this equation and the equity premium derived before using the no-arbitrage condition. The second-order moment is derived,

$$\begin{aligned}
VAR_t^Q(\tilde{r}_{t+1}^i) &= E_t^Q \left( (\tilde{r}_{t+1}^i)^2 \right) - \left( E_t^Q(\tilde{r}_{t+1}^i) \right)^2 \\
&= \frac{\partial^2 mgf_t^Q(\tilde{r}_{t+1}^i; \nu)}{\partial \nu^2} \Big|_{\nu=0} - \left( \frac{\partial mgf_t^Q(\tilde{r}_{t+1}^i; \nu)}{\partial \nu} \Big|_{\nu=0} \right)^2 \\
&= \left( \frac{\sigma_p(\tilde{r}^i)}{1 - \sigma_p(\tilde{\mathbf{m}}_1)} \right)^2 p_t + \left( \frac{\sigma_n(\tilde{r}^i)}{1 - \sigma_n(\tilde{\mathbf{m}}_1)} \right)^2 n_t + \left( \frac{\sigma_{lp}(\tilde{r}^i)}{1 - \sigma_{lp}(\tilde{\mathbf{m}}_1)} \right)^2 lp_t + \left( \frac{\sigma_q(\tilde{r}^i)}{1 - \sigma_q(\tilde{\mathbf{m}}_1)} \right)^2 qt \\
&+ \underbrace{\left( \frac{\sigma_{ln}(\tilde{r}^i)}{1 - \sigma_{ln}(\tilde{\mathbf{m}}_1)} \right)^2 \overline{ln} + \tilde{r}^{i'} \mathbf{S}_1 \Sigma^{other} \mathbf{S}'_1 \tilde{r}^i + \sigma_i^2}_{\equiv C(Q^i)}.
\end{aligned} \tag{B.60}$$

## C Variables and parameters

Table C.1: Variables. (In order of first appearance)

Symbol	
$C_t$	consumption level
$Q_t$	the relative risk aversion state (RRA) variable
$m_t$	log real pricing kernel
$c_t$	$\ln(C_t)$
$q_t$	$\ln(Q_t)$
$\Delta c_t$	log change in consumption
$\Delta q_t$	log change in RRA of per period utility of the representative agent
$H_t$	external habit level (as in Campbell and Cochrane, 1999)
$\theta_t$	log change in the real industrial production index, or growth
$p_t$	upside macroeconomic uncertainty state variable, or “good” uncertainty, or shape parameter of the upside macroeconomic shock

$n_t$	downside macroeconomic uncertainty state variable, or “bad” uncertainty, or shape parameter of the downside macroeconomic shock
$u_t^\theta$	growth disturbance
$\omega_{p,t}$	upside macroeconomic shock
$\omega_{n,t}$	downside macroeconomic shock
$\mathbf{Y}_t^{mac}$	macroeconomic state variables consisting of $\{\theta_t, p_t, n_t\}$
$l_t$	log corporate bond loss rate
$u_t^l$	loss rate-specific shock
$\omega_{lp,t}$	upside loss rate (cash flow) shock
$\omega_{ln,t}$	downside loss rate (cash flow) shock
$lp_t$	upside loss rate (cash flow) uncertainty state variable, or shape parameter of the upside loss rate shock
$\mathbf{Y}_t^{fin}$	financial state variables consisting of $\{l_t, lp_t\}$
$g_t$	change in log earnings
$u_t^g$	earnings growth-specific disturbance
$\omega_{g,t}$	standardized earnings growth-specific shock
$\kappa_t$	log consumption-earnings ratio
$u_t^\kappa$	consumption-earnings ratio-specific disturbance
$\omega_{\kappa,t}$	standardized consumption-earnings ratio-specific shock
$\eta_t$	log dividend payout ratio
$u_t^\eta$	dividend payout ratio-specific disturbance
$\omega_{\eta,t}$	standardized dividend payout ratio-specific shock
$\Delta d_t$	log change in dividend
$u_t^q$	risk aversion-specific disturbance
$\omega_{q,t}$	risk aversion shock
$\pi_t$	inflation
$u_t^\pi$	inflation-specific disturbance
$\omega_{\pi,t}$	standardized inflation-specific shock
$\mathbf{Y}_t^{other}$	a vector of non-macro state variables, $[\pi_t, l_t, g_t, \kappa_t, \eta_t, lp_t, q_t]'$
$\mathbf{Y}_t$	a vector of all 10 state variables, $[\mathbf{Y}_t^{mac}, \mathbf{Y}_t^{fin}]'$
$\omega_t$	a vector of 9 independent shocks, $[\omega_{p,t}, \omega_{n,t}, \omega_{\pi,t}, \omega_{lp,t}, \omega_{ln,t}, \omega_{g,t}, \omega_{\kappa,t}, \omega_{\eta,t}, \omega_{q,t}]'$
$\tilde{m}_t$	log nominal pricing kernel
$r_t^f$	nominal risk free rate
$pc_t^1$	log price-coupon ratio of one period defaultable bond portfolio
$pc_t^N$	log price-coupon ratio of N-period defaultable bond portfolio
$PD_t$	price-dividend ratio
$\tilde{r}_t^i$	log nominal asset return for asset $i$ , $i \in \{eq, cb\}$
$E_t(r_{t+1}^i)$	expected return for asset $i$
$RP_t^i$	model-implied one-month expected excess returns for asset $i$
$VAR_t^i \equiv VAR_t(\tilde{r}_{t+1}^i)$	model-implied one-month expected physical variances for asset $i$
$VAR_t^{i,Q} \equiv VAR_t^Q(\tilde{r}_{t+1}^i)$	model-implied one-month expected risk-neutral variances for asset $i$
$RVAR_t^i$	empirical benchmark of one-month realized physical variances for asset $i$
$QVAR_t^{eq}$	empirical benchmark of one-month expected risk-neutral variances for equity
$E_t$	monthly earnings
$ra_t^{BEX}$	Bekaert-Engstrom-Xu’s Risk aversion
$unc_t^{BEX}$	Bekaert-Engstrom-Xu’s financial proxy to macroeconomic uncertainty

Table C.2: Parameters.

Symbol	Value
$\gamma$	2
$\bar{\theta}$	1.87584E-05
$\rho_\theta$	0.13100
$m_p$	1.39336E-05
$m_n$	-0.00020
$\bar{p}$	500
$\bar{n}$	16.14206
$\sigma_{\theta p}$	0.00011
$\sigma_{\theta n}$	0.00174
$\rho_p$	0.99968
$\rho_n$	0.91081

$\sigma_{pp}$	0.55277
$\sigma_{nn}$	2.17755
$l_0$	-0.00091
$\rho_{ll}$	0.83060
$m_{lp}$	0.00000
$m_{ln}$	0.00014
$\sigma_{lp}$	-4.36148E-06
$\sigma_{ln}$	0.00051
$\sigma_{llp}$	0.00060
$\sigma_{lln}$	0.00011
$\bar{l}_p$	5.21535
$\rho_{lp}$	0.85557
$\sigma_{lplp}$	1.86152
$\bar{l}_n$	103.57583
$g_0, \kappa_0, \eta_0, \pi_0$	0.0207, 0.1451, -0.0966, 0.0031
$\rho_{gg}, \rho_{\kappa\kappa}, \rho_{\eta\eta}, \rho_{\pi\pi}$	0.6589, 0.9303, 0.9102, 0.3973
$\rho_{g\theta}, \rho_{\kappa\theta}, \rho_{\eta\theta}, \rho_{\pi\theta}$	1.5181, -0.2731, 0.1765, -0.0718
$\rho_{gp}, \rho_{\kappa p}, \rho_{\eta p}, \rho_{\pi p}$	-5.13E-05, 4.54E-05, -3.24E-05, -2.07E-06
$\rho_{gn}, \rho_{\kappa n}, \rho_{\eta n}, \rho_{\pi n}$	0.0005, 0.0031, 0.0037, -7.04E-05
$\rho_{gl}, \rho_{\kappa l}, \rho_{\eta l}$	-1.2318, 2.3791, 2.1250
$\rho_{glp}, \rho_{\kappa lp}, \rho_{\eta lp}$	0.0007, -0.0008, -0.0008
$\sigma_{gp}, \sigma_{\kappa p}, \sigma_{\eta p}, \sigma_{\pi p}$	-8.52E-05, 6.30E-05, 6.21E-05, -4.83E-06
$\sigma_{gn}, \sigma_{\kappa n}, \sigma_{\eta n}, \sigma_{\pi n}$	-0.0033, 0.0066, 0.0068, 8.27E-05
$\sigma_{glp}, \sigma_{\kappa lp}, \sigma_{\eta lp}$	-0.0005, 0.0008, 0.0008
$\sigma_{gln}, \sigma_{\kappa ln}, \sigma_{\eta ln}$	-0.0002, 0.0004, 0.0004
$\sigma_{gg}, \sigma_{\kappa\kappa}, \sigma_{\eta\eta}, \sigma_{\pi\pi}$	0.0462, 0.0558, 0.0574, 0.0023
$q_0$	-0.0503
$\rho_{qq}$	0.7387
$\rho_{qp}$	0.0003
$\rho_{qn}$	0.0036
$\sigma_{qp}$	0.0004
$\sigma_{qn}$	0.0004
$\sigma_{qq}$	-0.0002
$\sigma_{q\kappa}$	-0.0040
$\sigma_{qq}$	0.1417
$\boldsymbol{\mu}$	(A.2)
$\mathbf{A}$	(A.3)
$\boldsymbol{\Sigma}$	(A.4)
$m_0$	(B.1)
$m_1$	(B.1)
$m_2$	(B.3)
$\tilde{m}_0$	(B.16)
$\tilde{m}_1$	(B.16)
$\tilde{m}_2$	(B.18)
$b_0^N$	(B.21)
$b_1^N$	(B.21)
$\tilde{\xi}_0^i$	(B.36)
$\tilde{\xi}_1^i$	(B.36)
$\tilde{r}^i$	(B.37)
$\sigma_i$	(B.37)
$\chi$	Table 3
$\chi^{unc}$	Table 9
$\hat{X}$	-

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## D Constructing Realized Speculative Corporate Bond Return Variances, $rvarcbSPEC$

The realized variance  $rvarcbSPEC$  at the daily frequency is the sum of squares of daily returns over the past 22 days; its monthly measure is obtained using the end of the month value. The daily speculative corporate

bond return is the log change in the daily series “ICE BofAML US High Yield Total Return Index” (source: FRED). Because the daily index only starts in February 1990, we use an empirical model to fill in the missing data from June 1986 to January 1990 (i.e., the sample of our paper spans the period from June 1986 to February 2015). To impute these missing data, we first obtain the best linear model of daily  $rvarcbSPEC$  given the goodness of fit criteria (BIC) and sample after 1990, and then use the model to impute  $rvarcbSPEC$  before 1990 using using daily observables. In the model selection, we use contemporaneous values and one and two period lagged values of the following observables: realized equity return variance, realized treasury bond return variance, realized all-market corporate bond return variance, and realized variance of 22-day daily changes in credit spread. Daily Treasury bond return is the log change in 10-year log Treasury bond market total return index (source: DataStream); other source data are discussed in Section 4. The final model is shown as below:

Table D.1: The empirical model to impute  $rvarcbSPEC$  from June 1986 to January 1990.

Constant	<b>-0.0001</b> (2.86E-05)
Equity $rvareq_t$	<b>0.0813</b> (0.0049)
Equity $rvareq_{t-1}$	<b>-0.0177</b> (0.0045)
TB $rvarbt_t$	<b>-0.3264</b> (0.0668)
TB $rvarbt_{t-1}$	<b>0.2714</b> (0.0708)
Corporate Bond $rvarcb_{t-1}$	<b>0.3585</b> (0.0770)
Credit Spread $rvarcsprd_t$	<b>0.0057</b> (0.0026)
$R^2$	79.4%
Correlation with Fitted	89.1%

## E Cash Flow Dynamics

In terms of data for cash flow processes, real earnings growth ( $g$ ), is defined as the change in log real earnings per capita. Real earnings is the product of real earnings per share and the number of shares outstanding during the same month. The log consumption-earnings ratio ( $\kappa$ ), uses real consumption and real earnings. Real monthly consumption is defined as the sum of seasonally-adjusted real personal consumption expenditures on nondurable goods and services; as widely recognized in the literature, the consumption deflator is different from the CPI and is computed using monthly data. The log dividend payout ratio ( $\eta$ ) is the log ratio of real dividends and real earnings. Therefore, consumption growth (dividend growth) is implicitly defined given  $g$  and  $\kappa$  ( $g$  and  $\eta$ ). Inflation ( $\pi$ ), is defined as the change in the log of the consumer price index (CPI) obtained from the Bureau of Labor Statistics (BLS).

The source for the consumption data is the U.S. Bureau of Economic Analysis (BEA); for the dividends and earnings data, it is Robert Shiller’s website. We use the 12-month trailing dividends and earnings, e.g.,  $E_t^{12} = E_{t-12} + \dots + E_{t-1}$  where  $E_t$  denotes monthly earnings. There are no true monthly earnings data because almost all firms report earnings results only quarterly. According to Shiller’s website, the monthly dividend and earnings data provided are inferred from the S&P four-quarter totals, which are available since 1926. Calculating 12-month trailing values of earnings and dividends is common practice to control for the strong seasonality in the data. Total market shares are obtained from CRSP. To obtain per capita units, we divide real consumption and real earnings by the population numbers provided by BEA.

The results are tabulated in Table E.1. Earnings growth is less persistent than the two equity yield variables, but loads positively and significantly on industrial production growth. The  $n_t$  state variable has a positive effect on the conditional mean of the consumption-earnings and dividend-earnings ratio, indicating that in recessions these ratios are expected to be larger than in normal times. This makes economic sense as consumption and dividends are likely smoothed over the cycle whereas earnings are particularly cycle sensitive (see also Longstaff and Piazzesi, 2004). Yet, the cyclicity of earnings growth does not show through a significant effect of  $n_t$  but rather appears through its positive dependence on industrial production growth directly and its negative dependence on the loss rate. Again, the ratio variables load significantly, but positively on the loss rate. The same intuition explains why the ratio variables load positively on  $\omega_n$  shocks and earnings growth loads negatively on this shock. The  $\omega_p$  and  $\omega_{lp}$  shocks do not have a significant effect on these state variables.

The projections implicitly define the residuals shock for the cash flow variables, which we found to be homoskedastic (in unreported results). These shocks still feature substantial and significant variability, and

are quite correlated. Essentially, because earnings growth is quite variable, the ratio variables are positively correlated with one another and negatively correlated with earnings growth. When pricing assets with the model, this correlation structure must be accounted for (see below). The correlations with the other state variable shocks and between these state variable shocks ( $\omega_p, \omega_n, \omega_{lp}, \omega_{ln}$ ) ought to be zero in theory and they are economically indeed close to zero. We report these correlations in Table E.2.

Table E.1: The Dynamics of Other Cash Flow Variables

This table shows the projection results of other cash flow dynamics. The dynamic processes of the pure cash flow variables (log earnings growth,  $g_{t+1}$ ; log consumption-earnings ratio,  $\kappa_{t+1}$ ; log dividend-earnings ratio,  $\eta_{t+1}$ ; inflation rate,  $\pi_{t+1}$ ) are shown in Section 2. These coefficients are estimated using simple linear projections. Robust standard errors are shown in parentheses. The adjusted  $R^2$  of the conditional mean part (with information set  $t$ ) is reported in the last row. Bold (italic) coefficients have  $<5\%$  ( $10\%$ ) p-values. The sample period is 1986/06 to 2015/02 (345 months).

	Earnings Growth	Log CE	Log DE	Inflation
	$g_{t+1}$	$\kappa_{t+1}$	$\eta_{t+1}$	$\pi_{t+1}$
Constant	0.0207 (0.0277)	<b>0.1451</b> (0.0459)	<b>-0.0966</b> (0.0340)	<b>0.0031</b> (0.0014)
AR	<b>0.6589</b> (0.0433)	<b>0.9303</b> (0.0095)	<b>0.9102</b> (0.0109)	<b>0.3973</b> (0.0500)
$\theta_t$	<b>1.5181</b> (0.6933)	-0.2731 (0.8565)	0.1765 (0.8880)	<b>-0.0718</b> (0.0315)
$p_t$	-5.13E-05 (5.70E-05)	4.54E-05 (7.00E-05)	-3.24E-05 (7.40E-05)	-2.07E-06 (2.84E-06)
$n_t$	0.0005 (0.0007)	<b>0.0031</b> (0.0008)	<b>0.0037</b> (0.0008)	<b>-7.04E-05</b> (1.94E-05)
$l_t$	<i>-1.2318</i> (0.7376)	<b>2.3791</b> (0.9352)	<b>2.1250</b> (0.9596)	
$lp_t$	<i>0.0007</i> (0.0004)	<i>-0.0008</i> (0.0004)	<i>-0.0008</i> (0.0004)	
$\omega_{p,t+1}$	-8.52E-05 (1.16E-04)	6.30E-05 (1.40E-04)	6.21E-05 (1.44E-04)	-4.83E-06 (5.87E-06)
$\omega_{n,t+1}$	<b>-0.0033</b> (0.0011)	<b>0.0066</b> (0.0013)	<b>0.0068</b> (0.0014)	8.27E-05 (5.60E-05)
$\omega_{lp,t+1}$	-0.0005 (0.0010)	0.0008 (0.0013)	0.0008 (0.0013)	
$\omega_{ln,t+1}$	<i>-0.0002</i> (0.0001)	<b>0.0004</b> (0.0001)	<b>0.0004</b> (0.0002)	
Gaussian shock volatility	<b>0.0462</b> (0.0018)	<b>0.0558</b> (0.0021)	<b>0.0574</b> (0.0022)	<b>0.0023</b> (0.0001)
Adjusted $R^2$ (conditional mean)	56.76%	98.34%	98.06%	20.85%

Table E.2: Filtered Shock Correlation Matrix

The table provides a correlation matrix of the shock structure of the economy. The shocks are summarized as follows:

$\omega_{p,t+1}$ :	upside (good) macroeconomic uncertainty shock	$\tilde{\Gamma}(p_t, 1)$ ;
$\omega_{n,t+1}$ :	downside (bad) macroeconomic uncertainty shock	$\tilde{\Gamma}(n_t, 1)$ ;
$\omega_{lp,t+1}$ :	upside loss rate uncertainty shock	$\tilde{\Gamma}(lp_t, 1)$ ;
$\omega_{ln,t+1}$ :	downside loss rate uncertainty shock	$\tilde{\Gamma}(\bar{ln}, 1)$ ;
$\omega_{g,t+1}$ :	log earnings growth-specific shock	$N(0,1)$ ;
$\omega_{\kappa,t+1}$ :	log C/E-specific shock	$N(0,1)$ ;
$\omega_{\eta,t+1}$ :	log D/E-specific shock	$N(0,1)$ ;
$\omega_{q,t+1}$ :	risk aversion-specific shock	$\tilde{\Gamma}(q_t, 1)$ .

Bold (italic) coefficients have <5% (10%) p-values. The sample period is 1986/06 to 2015/02 (345 months).

	$\omega_p$	$\omega_n$	$\omega_{\pi}$	$\omega_{lp}$	$\omega_{ln}$	$\omega_g$	$\omega_{\kappa}$	$\omega_{\eta}$	$\omega_q$
$\omega_p$	1	<b>-0.1129</b>	0.0000	-0.0011	-0.0007	0.0000	0.0000	0.0000	0.0000
$\omega_n$		1	0.0000	<i>-0.0912</i>	-0.0828	0.0000	0.0000	0.0000	0.0000
$\omega_{\pi}$			1	<i>0.0943</i>	-0.0442	<b>0.1060</b>	-0.0120	-0.0536	0.0360
$\omega_{lp}$				1	<b>-0.1877</b>	0.0000	0.0000	0.0000	0.0296
$\omega_{ln}$					1	0.0000	0.0000	0.0000	<b>-0.1789</b>
$\omega_g$						1	<b>-0.6765</b>	<b>-0.6589</b>	0.0613
$\omega_{\kappa}$							1	<b>0.9863</b>	-0.0413
$\omega_{\eta}$								1	-0.0351
$\omega_q$									1

## F Supplementary Tables and Figures

Table F.1: Summary Statistics of Financial Instruments Spanning Risk Aversion

This table presents summary statistics of the 6 financial instruments that are used to span our risk aversion measure: “tsprd” is the difference between 10-year treasury yield and 3-month Treasury yield; “csprd” is the difference between Moody’s Baa yield and the 10-year zero-coupon Treasury yield; “EY5yr” (“DY5yr”) is the detrended earnings (dividend) yield where the moving average takes the 5 year average of monthly earnings yield, starting one year before; “rvareq” and “rvarcb” are realized variances of log equity returns and log corporate bond returns, calculated from daily returns; “qvareq” is the risk-neutral conditional variance of log equity returns; for the early years (before 1990), we use VXO and authors’ calculations. Bold (italic) coefficients have <5% (10%) p-values. Block bootstrapped errors are shown in parentheses. The sample period is from 1986/06 to 2015/02 (345 months).

	tsprd	csprd	DY5yr	EY5yr	rvareq	qvareq	rvarcb
Correlation Matrix							
tsprd	1	<b>0.3524</b>	<b>0.2595</b>	<b>0.2526</b>	<b>0.1269</b>	<b>0.1244</b>	<b>0.2952</b>
csprd		1	<b>0.4990</b>	<b>0.5083</b>	<b>0.4786</b>	<b>0.5988</b>	<b>0.5330</b>
DY5yr			1	<b>0.8966</b>	<b>0.1678</b>	<b>0.1650</b>	<b>0.3101</b>
EY5yr				1	<b>0.1399</b>	<b>0.1564</b>	<b>0.3359</b>
rvareq					1	<b>0.8431</b>	<b>0.5943</b>
qvareq						1	<b>0.5376</b>
rvarcb							1
Summary Statistics							
Mean	<b>0.0179</b>	<b>0.0231</b>	<b>-0.0030</b>	<b>-0.0074</b>	<b>0.0029</b>	<b>0.0040</b>	<b>0.0002</b>
Boot.SE	(0.0006)	(0.0004)	(0.0003)	(0.0008)	(0.0003)	(0.0002)	(0.0000)
S.D.	<b>0.0116</b>	<b>0.0075</b>	<b>0.0061</b>	<b>0.0149</b>	<b>0.0059</b>	<b>0.0037</b>	<b>0.0003</b>
Boot.SE	(0.0003)	(0.0005)	(0.0003)	(0.0007)	(0.0014)	(0.0005)	(0.0000)
Skewness	<b>-0.2322</b>	<b>1.7891</b>	0.0959	<b>-0.3495</b>	<b>8.1198</b>	<b>3.7225</b>	<b>4.2227</b>
Boot.SE	(0.0810)	(0.2515)	(0.1882)	(0.1502)	(1.5951)	(0.5123)	(0.6872)
AR(1)	<b>0.9668</b>	<b>0.9640</b>	<b>0.9822</b>	<b>0.9843</b>	<b>0.4312</b>	<b>0.7462</b>	<b>0.5775</b>
SE	(0.0137)	(0.0143)	(0.0083)	(0.0068)	(0.0488)	(0.0360)	(0.0441)

Table F.2: On the Predictive Power of Risk Aversion and Uncertainty for Future Consumption Growth

This table reports the coefficient estimates of the following predictive regression,

$$\frac{1}{k} \sum_{\tau=1}^k \Delta c_{t+\tau} = a_k + \mathbf{b}'_k \mathbf{x}_t + \epsilon_{t+k},$$

where  $\Delta c_{t+\tau}$  is the 1-month log consumption growth from  $t + \tau - 1$  to  $t + \tau$  and  $x_t$  indicates risk measures and  $\mathbf{x}_t$  represents a vector of current predictors: (1) our financial instrument proxy of economic uncertainty,  $unc^{BEX}$ , (2) our financial instrument proxy of risk aversion,  $ra^{BEX}$ , (3) the risk-neutral conditional variance (the square of the month-end VIX (after 1990) / VXO (prior to 1990) index divided by 120000),  $QVAR$ , and (4) the true total macroeconomic uncertainty filtered from industrial production growth  $unc^{true}$ . The coefficients are scaled by the standard deviation of the predictor in the same column for interpretation purposes. Hodrick (1992) standard errors are reported in parentheses, and adjusted  $R^2$ s are in %. Bold (italic) coefficients have  $<5\%$  ( $10\%$ ) p-values. This table relates to Table 10 which predicts actual output growth.

	$unc^{true}$	$ra^{BEX}$	
A. Univariate			
1m	<b>-0.0005</b> (0.0001) 3.7%	<b>-0.0005</b> (0.0001) 3.3%	
3m	<b>-0.0005</b> (0.0001) 13.1%	<b>-0.0004</b> (0.0001) 8.8%	
12m	<b>-0.0004</b> (0.0001) 12.8%	<b>-0.0003</b> (0.0001) 6.1%	
B. Multivariate			$R^2$
1m	<b>-0.0003</b> (0.0002)	-0.0003 (0.0002)	4.4%
3m	<b>-0.0004</b> (0.0002)	-0.0002 (0.0002)	14.1%
12m	<b>-0.0003</b> (0.0001)	-0.0001 (0.0001)	12.9%

Table F.3: Projecting Pure Cash Flow Uncertainty using OLS

This table presents regression results of the estimated monthly pure cash flow uncertainty (from loss rate) on a set of monthly asset prices; some are used to span the time-varying risk aversion. The dependent variable is  $lp_t$ , the time-varying shape parameter of the pure right-tail loss rate residual (after controlling for macroeconomic shocks) as demonstrated in Table 2.  $\times 10^{-3}$  in the header means that the coefficients and their SEs reported are divided by 1000 for reporting convenience. “VARC” reports the variance decomposition. Bold (italic) coefficients have  $<5\%$  ( $10\%$ ) p-values. Robust and efficient standard errors are shown in parentheses. Adjusted  $R^2$ s are reported. The sample period is 1986/06 to 2015/02 (345 months).

	$(\times 10^{-3})$	
	$lp_t$	VARC
constant	<i>-0.001</i> (0.001)	
$\chi_{tsprd}$	<b>-0.058</b> (0.011)	-2.33%
$\chi_{csprd}$	<b>0.202</b> (0.025)	62.69%
$\chi_{DY5yr}$	<b>0.234</b> (0.046)	41.57%
$\chi_{EY5yr}$	<b>-0.061</b> (0.019)	-22.57%
$\chi_{rvareq}$	-0.026 (0.062)	-3.76%
$\chi_{qvar eq}$	<i>0.119</i> (0.067)	13.25%
$\chi_{rvarc b}$	<b>1.779</b> (0.593)	13.67%
$\chi_{rvarc b SPEC}$	-0.223 (0.556)	-2.51%
$R^2$	9.11%	

Table F.4: External validation (1): Risk aversion and macro announcement shocks

This table reports the regression results of our risk aversion index on cumulatively monthly macro announcement shocks of 7 variables, i.e., coefficient “c”. The regression framework is  $ra_t^{BEX} = a + b \times ra_{t-1}^{BEX} + c \times MacroShocks_t + \epsilon_t$ . We thank Marie Hoerova for providing the macro announcement shock data; the macro shocks are available from 2000 to 2015. In Panel B, we report the variance decomposition among the 7 shocks.

A: Univariate regression				
Variables:	Est:	(SE):	t:	$R^2$
AR(1) + No macro shock				67.3%
AR(1) + IP (industrial production)	-0.132	(0.034)	-3.898	69.7%
AR(1) + UR (unemployment rate)	0.080	(0.035)	2.295	68.2%
AR(1) + GDP	0.064	(0.035)	1.837	67.9%
AR(1) + CPI (consumer price index)	0.045	(0.035)	1.278	67.6%
AR(1) + BOP (balance of payments)	0.001	(0.035)	0.032	67.3%
AR(1) + CC (consumer confidence)	-0.022	(0.035)	-0.616	67.4%
AR(1) + MC (manufacturing confidence)	-0.038	(0.035)	-1.089	67.5%
B: Multivariate regression ( $R^2=71.7\%$ )				
Variables:	Est:	(SE):	t:	VARC:
AR(1)	0.810	(0.040)	20.204	
IP (industrial production)	-0.132	(0.033)	-3.961	50%
UR (unemployment rate)	0.074	(0.034)	2.214	33%
GDP	0.074	(0.033)	2.205	12%
CPI (consumer price index)	0.032	(0.034)	0.927	-3%
BOP (balance of payments)	0.018	(0.034)	0.526	1%
CC (consumer confidence)	-0.016	(0.034)	-0.472	-1%
MC (manufacturing confidence)	-0.039	(0.033)	-1.168	8%

Table F.5: External validation (2): Regression evidence

This table reports the regression coefficients of our risk aversion index on standardized (left panel) or standardized+UC-orthogonalized (right panel) sentiment measures. The 16 external measures of sentiment or confidence are introduced in Table 11.

		Raw, Z				Uncertainty-orthogonalized, Z			
		Est	SE	t	R2	Est	SE	t	R2
1	<i>CB_CC</i>	<b>-0.194</b>	(0.033)	-5.87	8%	<b>-0.129</b>	(0.034)	-3.81	3%
2	<i>Michigan_Sent</i>	<b>-0.249</b>	(0.032)	-7.75	13%	<b>-0.156</b>	(0.034)	-4.66	5%
3	<i>OECD_CC</i>	<b>-0.296</b>	(0.031)	-9.51	18%	<b>-0.105</b>	(0.034)	-3.08	2%
4	<i>IPSOS_Sent</i>	<b>-0.439</b>	(0.048)	-9.14	27%	<b>-0.328</b>	(0.052)	-6.33	15%
5	<i>DEG_fears25</i>	-0.119	(0.075)	-1.59	1%	-0.096	(0.075)	-1.28	0%
6	<i>DEG_fears30</i>	-0.119	(0.075)	-1.59	1%	-0.094	(0.075)	-1.25	0%
7	<i>Yale_crashC</i>	<b>-0.406</b>	(0.046)	-8.74	25%	<b>-0.231</b>	(0.051)	-4.48	8%
8	<i>Yale_valuationC</i>	<b>0.293</b>	(0.050)	5.84	13%	<b>0.203</b>	(0.052)	3.90	6%
9	<i>AAII_bullish</i>	<b>-0.080</b>	(0.035)	-2.27	1%	-0.049	(0.035)	-1.38	0%
10	<i>AAII_bearish</i>	<b>0.226</b>	(0.034)	6.71	10%	<b>0.145</b>	(0.035)	4.17	4%
11	<i>Sentix_Sent</i>	<b>-0.555</b>	(0.044)	-12.56	43%	<b>-0.358</b>	(0.053)	-6.74	18%
12	<i>OECD_BC</i>	<b>-0.252</b>	(0.032)	-7.84	13%	<b>-0.156</b>	(0.034)	-4.66	5%
13	<i>SFFed_NewsSent</i>	<b>-0.340</b>	(0.030)	-11.32	24%	<b>-0.218</b>	(0.033)	-6.66	10%
14	<i>BW_Sent_ORTH</i>	<b>-0.110</b>	(0.034)	-3.21	2%	<b>-0.097</b>	(0.034)	-2.83	2%
15	<i>CreditSuisse_RiskAppetite</i>	<b>-0.382</b>	(0.042)	-9.14	24%	<b>-0.219</b>	(0.046)	-4.76	8%
16	<i>habit_RiskAversion</i>	<b>0.151</b>	(0.039)	3.90	4%	<b>0.125</b>	(0.039)	3.19	3%

Table F.6: External validation (2): Risk aversion and PCA sentiment measures

This table focuses on the two major groups of external sentiment measures, consumer and investor; see Table 11 for details on individual measures. Here is the summary for PCA measures:

	Consumer (1-6)	Investor (7-11)	Business (12)
ConsumerPC1-(1)	X		
ConsumerPC1-(2)	X		X
InvestorPC1-(1)		X	
InvestorPC1-(2)		X	X

We obtain the 1st PC of the adjusted (standardized, uncertainty-orthogonalized, sign-corrected) individual measures in each group, consumer and investor; because business sentiment might potentially reflect both, we create an alternative consumer or investor PC1 measure incorporating business. We then run contemporaneous regressions of our risk aversion index on these PCA measures at the monthly frequency. “VARC” indicates variance decomposition. Bold correlation coefficients have <5% p-values.

DV:	Risk Aversion, $ra^{BEX}$						
ConsumerPC1-(1)	<b>0.304</b>				<b>0.203</b>	<b>0.172</b>	
	(0.045)				(0.059)	(0.080)	
VARC					58.2%	51.8%	
ConsumerPC1-(2)		<b>0.251</b>					<b>0.158</b>
		(0.039)					(0.051)
VARC							54.0%
InvestorPC1-(1)			<b>0.293</b>		<b>0.157</b>		<b>0.166</b>
			(0.047)		(0.060)		(0.062)
VARC					41.8%		46.0%
InvestorPC1-(2)				<b>0.273</b>		<b>0.145</b>	
				(0.041)		(0.072)	
VARC							48.2%
R2	22.9%	21.5%	20.4%	22.6%	30.5%	24.4%	24.7%

Table F.7: Daily risk variables during the COVID-19 crisis and the Global Financial Crisis (see continuous values in Figure 5)

	Daily Risk aversion $ra^{BEX}$				Daily Financial proxy, Uncertainty $unc^{BEX}$				Monthly Actual Uncertainty
	Mean	Vol	Skew	AR(1)	Mean	Vol	Skew	AR(1)	Data
Panel A. COVID-19 Crisis									
January,2020	2.67	0.07	1.51	0.68	1.98	0.06	1.31	0.83	2.65
February,2020	2.99	0.64	2.27	0.90	2.14	0.16	1.80	0.95	2.37
March,2020	10.06	6.55	1.32	0.38	3.73	0.77	-0.40	0.95	4.78
April,2020	4.85	1.04	0.66	0.73	3.67	0.59	0.03	0.99	7.52
May,2020	3.87	0.30	0.65	0.68	2.91	0.12	0.09	0.95	-
Panel B. Global Financial Crisis									
September,2008	4.03	0.75	1.62	0.65	3.13	0.26	0.78	0.94	5.27
October,2008	13.39	7.77	1.36	0.40	4.42	0.33	-0.34	0.82	3.88
November,2008	11.46	6.62	1.74	0.65	4.25	0.13	0.70	0.60	4.18
December,2008	6.95	2.11	1.82	0.86	4.17	0.09	1.50	0.81	4.99
January,2009	7.28	1.57	1.88	0.39	4.01	0.08	0.25	0.72	5.11
February,2009	7.01	0.98	1.55	0.54	3.97	0.09	0.23	0.84	4.56
March,2009	6.73	1.25	0.90	0.81	4.12	0.06	0.63	0.58	4.63
April,2009	5.15	0.30	0.63	0.53	3.82	0.12	-0.11	0.96	4.28
Panel C. Long Sample									
1990/1/1-2015/2/27	3.02	1.20	11.99	0.85	1.88	0.65	0.67	0.997	1.88
1990/1/1-2020/6/23	3.04	1.17	12.06	0.87	1.95	0.63	0.60	0.997	1.94

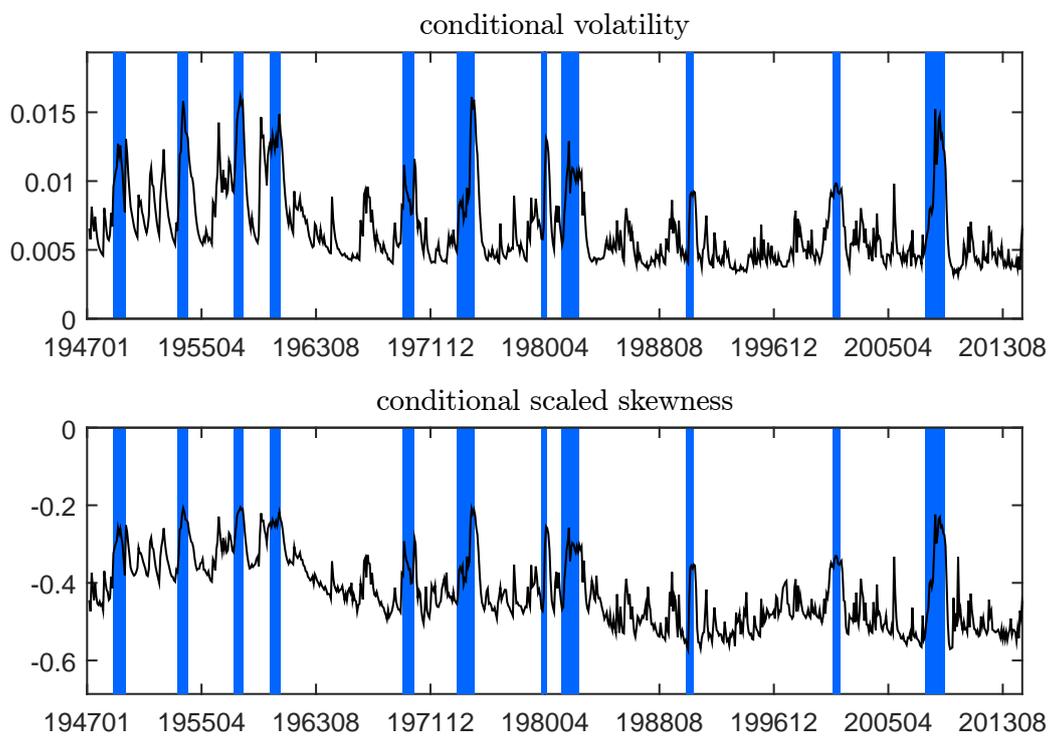


Figure F.1: Model-implied conditional moments of industrial production growth. The shaded regions are NBER recession months from the NBER website.

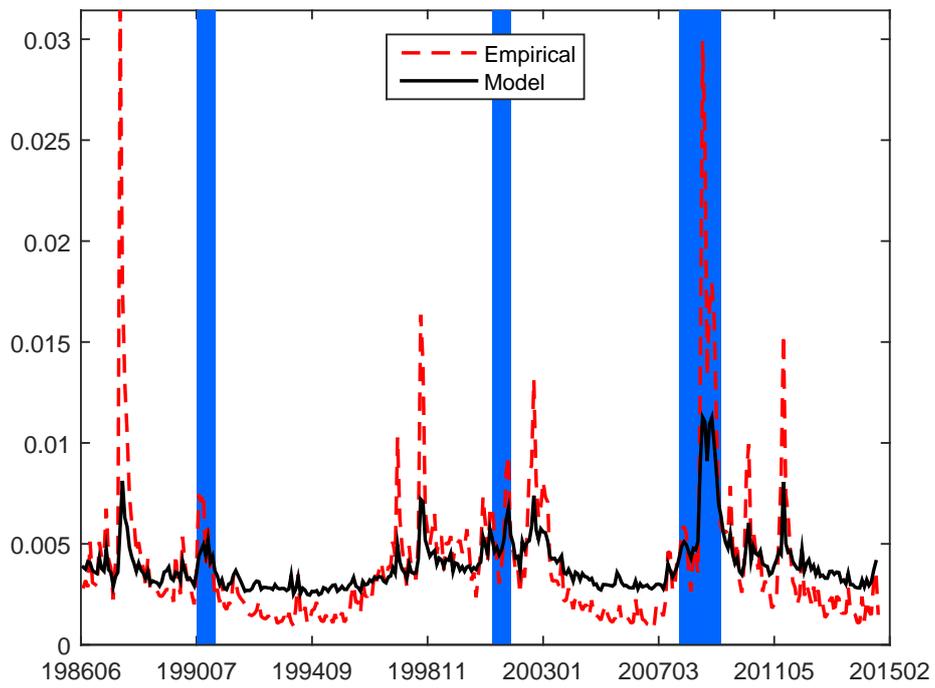


Figure F.2: Model-Implied and Empirical Risk-Neutral Conditional Equity Return Variances

The shaded regions are NBER recession months from the NBER website. The two series are 87.90% correlated.

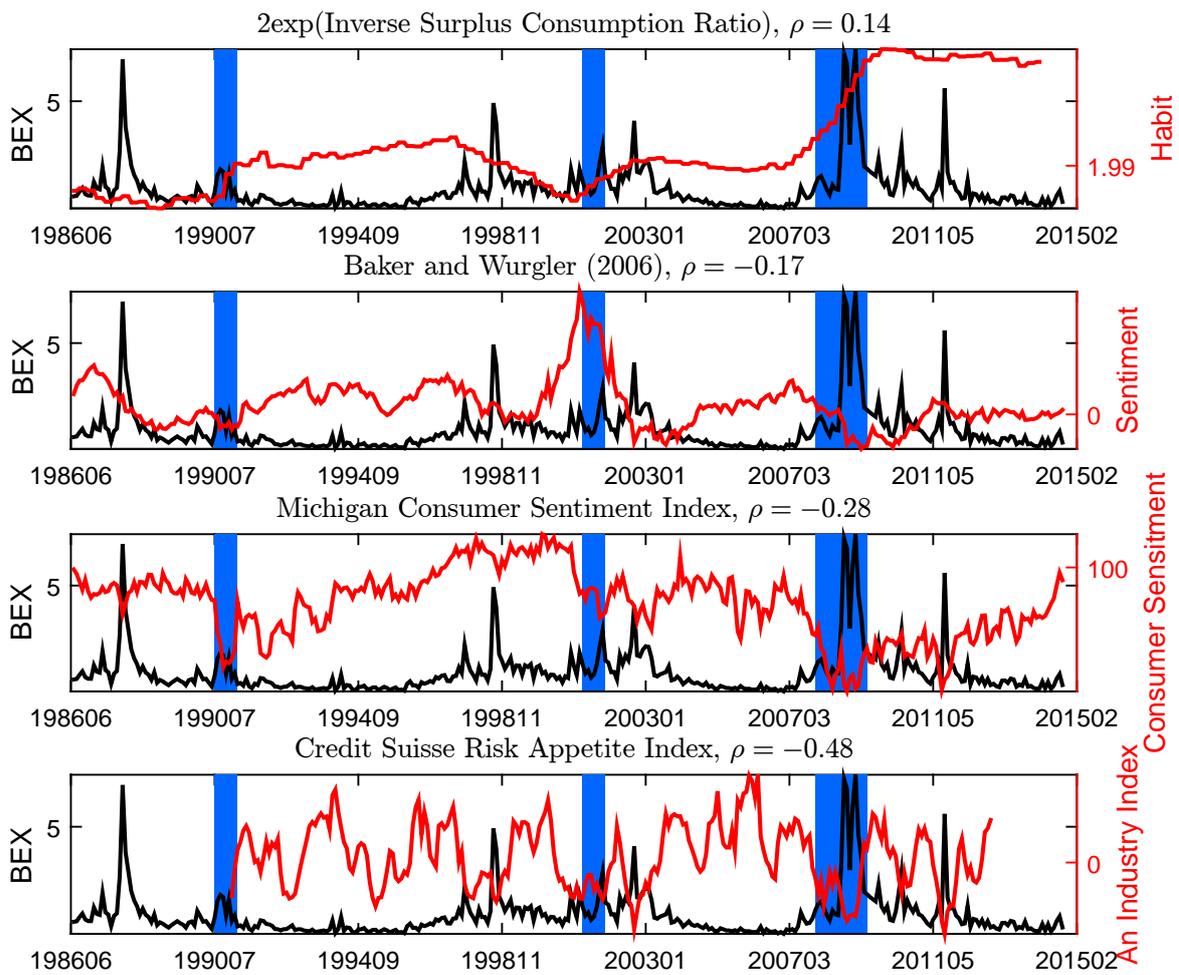


Figure F.3: Risk aversion/sentiment measures: A comparison.

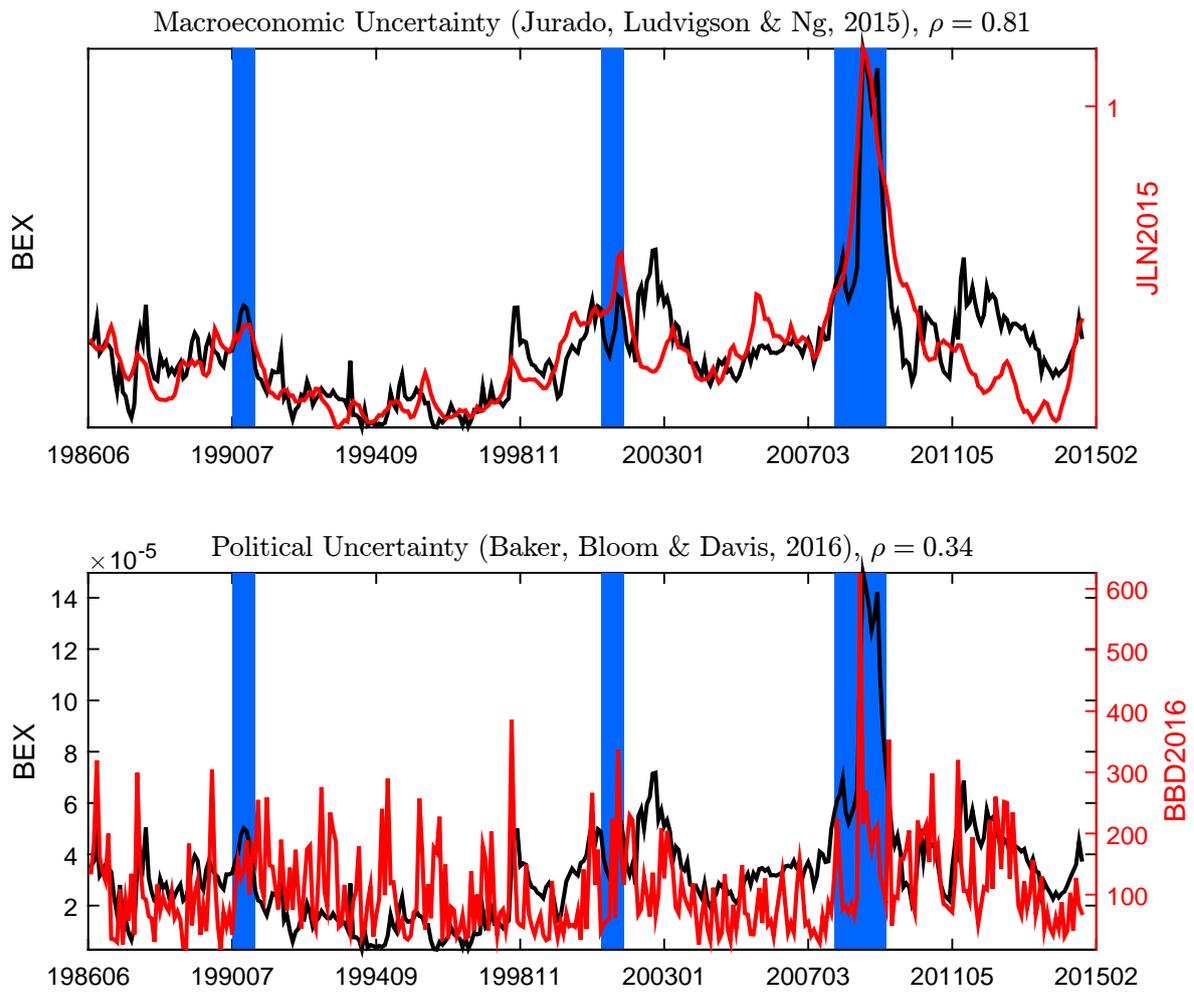


Figure F.4: Uncertainty measures: A comparison.

## G Robustness and Alternative Measures

The robustness checks are two fold. First, we explore the role of other model parameter choices in the estimation; for instance, the constant curvature parameter  $\gamma$  ( $=2$ ) as commonly imposed by the macro and asset pricing literature. Second, we explore the role of key modeling assumptions in the estimation; for instance, the role of  $p_t$  (time-varying or constant) and the  $q_t$  shock structure (see Equation (16)). For the constant  $p_t$  alternatives, the loss rate and  $q_t$  are re-estimated completely; model results and details are available upon request.

In summary, we report 6 sets of re-estimations of alternative  $q_t$ s. We organize the results into two general sets for comparison convenience:

1. The role of other model parameter choices:

Est(0). Fix  $\gamma = 2$  (the paper measure)

Est(1). Free up  $\gamma$

Est(2). Fix  $\gamma = 1.1$

Est(3). Fix  $\gamma = 3.5$

2. The role of key modeling assumptions:

Est(0). pt=time-varying; paper's  $q_t$  process

Est(4). pt=500;  $q_t$  only loads on  $\omega_{p,t}$  and  $\omega_{n,t}$

Est(5). pt=500; paper's  $q_t$  process

Est(6). pt=time-varying;  $q_t$  only loads on  $\omega_{p,t}$  and  $\omega_{n,t}$

Table G.1: Estimation results

	Est(0)	Est(1)	Est(2)	Est(3)	Est(4)	Est(5)	Est(6)
PAPER							
Free up $\gamma$		X					
Fix $p_t$ at 500					X	X	
$q_t$ loads on $\omega_{p,t}, \omega_{n,t}$	X	X	X	X	X	X	X
$q_t$ also loads on $\omega_{g,t}, \omega_{\kappa,t}$	X	X	X	X		X	
A. Efficient GMM Estimators							
constant	<b>0.050</b> (0.014)	<b>0.109</b> (0.010)	<b>0.081</b> (0.017)	-0.047 (0.187)	<b>-2.582</b> (1.405)	<b>-0.271</b> (0.014)	<b>0.059</b> (0.011)
$\chi_{tsprd}$	-0.753 (0.566)	0.525 (0.376)	1.199 (0.797)	<b>-2.329</b> (0.163)	-1.861 (1.220)	-0.972 (0.673)	0.222 (0.511)
$\chi_{csprd}$	<b>7.166</b> (1.030)	-1.034 (0.773)	<b>4.493</b> (0.913)	0.304 (0.214)	<b>9.412</b> (1.720)	<b>15.032</b> (0.825)	<b>1.948</b> (0.564)
$\chi_{EY5yr}$	<b>0.763</b> (0.291)	<b>0.939</b> (0.170)	0.286 (0.429)	<b>1.859</b> (0.165)	0.327 (0.684)	0.520 (0.647)	0.126 (0.265)
$\chi_{rvareq}$	<b>-16.984</b> (0.490)	<b>-11.877</b> (0.267)	<b>-32.314</b> (0.734)	<b>-7.272</b> (0.086)	<b>-41.643</b> (0.683)	<b>-45.372</b> (0.632)	<b>-20.195</b> (0.460)
$\chi_{qvarreq}$	<b>54.038</b> (1.753)	<b>34.881</b> (2.299)	<b>96.258</b> (3.044)	<b>19.869</b> (0.221)	<b>98.873</b> (2.514)	<b>103.111</b> (3.550)	<b>61.196</b> (1.551)
$\chi_{rvarcb}$	<b>118.248</b> (10.826)	<b>71.822</b> (7.290)	<b>146.512</b> (18.529)	<b>-40.139</b> (1.141)	<b>236.960</b> (19.544)	<b>228.853</b> (22.675)	<b>105.721</b> (11.500)
B. Other model parameter							
$\gamma$	2 -	<b>2.124</b> (0.104)	1.1 -	3.5 -	2 -	2 -	2 -
C. Correlation with the NBER Indicator							
$\rho(q_t, NBER_t)$	<b>0.454</b> (0.043)	<b>0.389</b> (0.046)	<b>0.408</b> (0.045)	<b>0.345</b> (0.047)	<b>0.426</b> (0.044)	<b>0.442</b> (0.043)	<b>0.401</b> (0.045)
D. Model Specifications							
Hansen's J	41.1254	38.6165	42.4700	48.9860	44.8785	41.4420	41.2904
p-value	0.0671	0.0873	0.0509	0.0116	0.0302	0.0630	0.0649
Rejected				X	X		

Table G.2: Fit of moments

		Est(0)	Est(1)	Est(2)	Est(3)	Emp. Av.	Boot.SE
		PAPER					
Mom 1	Equity Risk Premium	<b>0.00800</b>	<b>0.00658</b>	<b>0.00636</b>	<b>0.00653</b>	0.00530	(0.00246)
Mom 2	Equity Physical Variance	<b>0.00325</b>	<b>0.00372</b>	<b>0.00344</b>	<b>0.00289</b>	0.00286	(0.00051)
Mom 3	Equity Risk-neutral Variance	<b>0.00393</b>	<b>0.00446</b>	<b>0.00401</b>	<b>0.00326</b>	0.00397	(0.00049)
Mom 4	Corporate Bond Risk Premium	<i>0.00488</i>	<b>0.00332</b>	<b>0.00309</b>	0.01443	0.00388	(0.00050)
Mom 5	Corporate Bond Physical Variance	<b>0.00023</b>	<b>0.00023</b>	<b>0.00023</b>	0.00035	0.00024	(0.00003)
Mom 6	Risk Aversion Innovation Variance	<b>0.00783</b>	<b>0.00342</b>	<b>0.02511</b>	<b>0.00097</b>	0.00293	(0.00061)
Mom 7	Risk Aversion Innovation Unscaled Skewness	<b>0.00222</b>	<i>0.00088</i>	<b>0.01096</b>	<b>0.00019</b>	0.00041	(0.00022)
		Est(0)	Est(4)	Est(5)	Est(6)	Emp. Av.	Boot.SE
		PAPER					
Mom 1	Equity Risk Premium	<b>0.00800</b>	<b>0.00921</b>	<b>0.00986</b>	<b>0.00785</b>	0.00530	(0.00246)
Mom 2	Equity Physical Variance	<b>0.00325</b>	0.00409	<b>0.00382</b>	<b>0.00346</b>	0.00286	(0.00051)
Mom 3	Equity Risk-neutral Variance	<b>0.00393</b>	<i>0.00492</i>	<b>0.00462</b>	<b>0.00412</b>	0.00397	(0.00049)
Mom 4	Corporate Bond Risk Premium	<i>0.00488</i>	<b>0.00425</b>	<b>0.00396</b>	<b>0.00429</b>	0.00388	(0.00050)
Mom 5	Corporate Bond Physical Variance	<b>0.00023</b>	0.00032	0.00032	<b>0.00023</b>	0.00024	(0.00003)
Mom 6	Risk Aversion Innovation Variance	<b>0.00783</b>	<b>0.01804</b>	<b>0.02173</b>	<b>0.00767</b>	0.00293	(0.00061)
Mom 7	Risk Aversion Innovation Unscaled Skewness	<b>0.00222</b>	<b>0.00861</b>	<b>0.01029</b>	<b>0.00239</b>	0.00041	(0.00022)

Table G.3: Correlation among  $q_t$  alternatives

	Est(0)	Est(1)	Est(2)	Est(3)
$q_t, \text{Est}(0)$	1.0000	0.9636	0.9829	0.7195
$q_t, \text{Est}(1)$		1.0000	0.9873	0.7213
$q_t, \text{Est}(2)$			1.0000	0.7044
$q_t, \text{Est}(3)$				1.0000
	Est(0)	Est(4)	Est(5)	Est(6)
$q_t, \text{Est}(0)$	1.0000	0.9810	0.9812	0.9798
$q_t, \text{Est}(4)$		1.0000	0.9944	0.9824
$q_t, \text{Est}(5)$			1.0000	0.9681
$q_t, \text{Est}(6)$				1.0000

Table G.4: Cyclicalities of  $q_t$  alternatives and their VRP-residual

	Est(0)	Est(1)	Est(2)	Est(3)	Est(4)	Est(5)	Est(6)
	$q_{New,t} = a + b * VRP_t + \epsilon_t$						
b	<b>170.507</b>	<b>84.544</b>	<b>255.588</b>	<b>41.354</b>	<b>260.589</b>	<b>286.996</b>	<b>160.401</b>
	(3.474)	(1.270)	(3.121)	(1.946)	(4.711)	(6.446)	(1.718)
R2	0.875	0.928	0.951	0.567	0.899	0.853	0.962
	$\epsilon_t = \alpha + \beta * NBER_t + v_t$						
$\beta$	<b>0.070</b>	<b>0.012</b>	<b>0.048</b>	<b>0.013</b>	<b>0.075</b>	<b>0.111</b>	<b>0.026</b>
	(0.011)	(0.004)	(0.010)	(0.006)	(0.015)	(0.020)	(0.005)
R2	0.108	0.0227	0.0635	0.00979	0.0707	0.0816	0.0614